Total: Name:

Math 126 **Third Exam** **Fall 2021**

Write neat, concise, and accurate solutions to each of the problems below—I will not give partial credit for steps I cannot follow. Include all relevant steps, use correct notation, give sufficient details, and finish problems with a complete sentence; either in words or with an appropriate equation. Each problem is worth six points.

1. Evaluate $\displaystyle\int \frac{3x+4}{x^2-5x+6}\,dx$.

use partial fractions

$$\frac{3x+4}{(x-2)(x-3)} = \frac{A}{x-3} + \frac{B}{x-2} \qquad x=3 \Rightarrow 13 = A$$

$$3x+4 = A(x-2) + B(x-3) \qquad x=2 \Rightarrow 10 = -B$$

$$\int \frac{3x+4}{x^2-5x+6}\,dx = \int \left(\frac{13}{x-3} - \frac{10}{x-2}\right) dx$$

$$= 13\ln|x-3| - 10\ln|x-2| + C$$

2. Let h be the function that satisfies $h'(t) = 60 - \frac{1}{4}h(t)$ and $h(0) = 450$. Find the function h and a value of t for which $h(t) = 300$.

$$(h(t)-240)' = -\tfrac{1}{4}(h(t)-240)$$

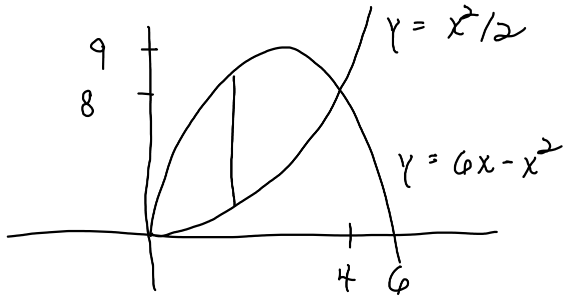
$$h(t) - 240 = 210\, e^{-t/4}$$

$$h(t) = 240 + 210\, e^{-t/4}$$

$$h(t) = 300 \Rightarrow e^{-t/4} = \frac{60}{210} \Rightarrow t = 4\ln\tfrac{7}{2}$$

The function $h(t)$ is $240 + 210\, e^{-t/4}$ and it equals 300 when $t = 4\ln 7/2$.

Let R be the region in the first quadrant that lies between the curves $y = \frac{1}{2}x^2$ and $y = 6x - x^2$. For problems 2–6, write an expression involving integrals that represents the requested quantity. Do **NOT** evaluate or simplify the integrals. Include a labeled graph of R in the space below.



3. the volume of the solid that is generated when R is revolved around the x-axis

$$V = \int_0^4 \left(\pi\left(6x - x^2\right)^2 - \pi\left(\frac{x^2}{2}\right)^2\right) dx$$

4. the volume of the solid that is generated when R is revolved around the y-axis

$$V = \int_0^4 2\pi x\left(6x - \frac{3}{2}x^2\right) dx$$

5. the volume of the solid that is generated when R is revolved around the line $x = 5$

$$V = \int_0^4 2\pi(5 - x)\left(6x - \frac{3}{2}x^2\right) dx$$

6. the volume of the solid whose base is R and each cross-section of the solid taken perpendicular to the x-axis is a semicircle

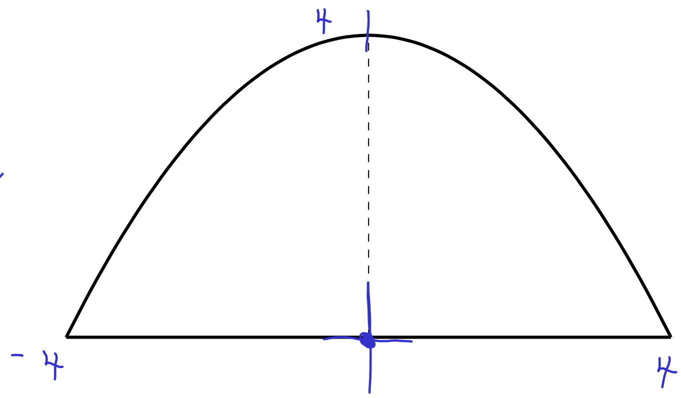
$r = \frac{1}{2}\left(6x - \frac{3}{2}x^2\right)$ $\qquad V = \int_0^4 \frac{\pi}{8}\left(6x - \frac{3}{2}x^2\right)^2 dx$

$\frac{\pi}{2} \cdot r^2$ is area

7. the x-coordinate of the center of mass of R

$$\bar{x} = \frac{\int_0^4 x\rho\left(6x - \frac{3}{2}x^2\right) dx}{\int_0^4 \rho\left(6x - \frac{3}{2}x^2\right) dx}$$

8. The figure below represents a large submerged window in the shape of a parabola. The base is eight feet, the height is four feet, and the top of the window is six feet below the surface of the water. Set up, BUT DO NOT EVALUATE, an integral that represents the total force of the water pushing against the window. Use w for the weight density of the water.



$y = 4 - kx^2$

$0 = 4 - k \cdot 4^2$

$k = \frac{1}{4}$

$y = 4 - \frac{1}{4}x^2$

$x^2 = 4(4-y)$

$x = \pm 2\sqrt{4-y}$

The force is $\int_0^4 w(10-y)\,4\sqrt{4-y}\,dy$

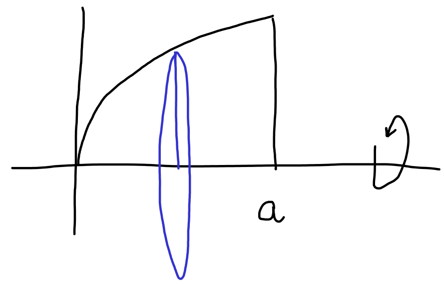
9. Evaluate $\displaystyle\int \frac{3x+5}{\sqrt{3-2x-x^2}}\,dx$.

$3 - 2x - x^2 = 4 - (x+1)^2$

$$\int \frac{3x+5}{\sqrt{3-2x-x^2}}\,dx = \int \left(\frac{3(x+1)}{\sqrt{4-(x+1)^2}} + \frac{2}{\sqrt{4-(x+1)^2}} \right) dx$$

$$= -3\sqrt{3-2x-x^2} + 2 \arcsin \frac{x+1}{2} + C$$

10. Let a be a positive constant, let R be the region that lies below the curve $y = \sqrt{x}$ and above the x-axis on the interval $[0, a]$, and let S be the solid that is generated when R is revolved around the x-axis. Assuming constant density ρ, the center of mass of S lies on the x-axis. Find the x-coordinate of this point (it will be a function of the parameter a).



$$\bar{x} = \frac{\int_0^a x \rho \pi (\sqrt{x})^2 \, dx}{\int_0^a \rho \pi (\sqrt{x})^2 \, dx}$$

$$= \frac{\int_0^a x^2 \, dx}{\int_0^a x \, dx}$$

$$= \frac{\frac{1}{3} a^3}{\frac{1}{2} a^2}$$

$$= \frac{2}{3} a$$

The x-coordinate of the center of mass is $\frac{2}{3} a$.