Vote Tallying

We discuss systems that declare a winner among some candidates, given the rankings of candidates by individual voters, and more generally, systems that produce an entire ranking of the candidates. For simplicity, we usually assume that voters are not allowed to give two candidates the same rank (but in the final ranking, ties may still be possible). If a single winner is to be declared, it will be the highest ranked candidate if the system produces an entire ranking. Ranking systems can also be used to fill more than one position from a single vote by choosing the top ranking candidates.

If there are only two candidates, the obvious method is "majority rules". This has no apparent problems, so we don't pay attention to 2-candidate contests, although systems that deal with more candidates can almost always be used for 2-candidate contests as well. If a system does not elect the majority winner in a 2-candidate race it is certainly suspect.

Properties of Voting Methods

Some general properties relevant to voting systems that seem desirable have been identified, studied, and named.

1. Condorcet winner: if a single candidate would beat every other candidate in a one on one contest, that winning candidate is called a Condorcet winner. In any particular contest there need not be a Condorcet winner, but if there is, then it seems reasonable to require that candidate to be ranked first.
2. Pareto property: if every voter prefers candidate A to candidate B then the final ranking should rank A above B.
3. Monotonicity: if one or more voters move candidate A up in their personal preference list then A should not move down in the final ranking.
4. Independence of Irrelevant Alternatives (IIA): if in the final ranking A is ranked higher than B, and if a third candidate C is removed from the contest and the ballots recounted then A should still be ranked higher than B.

It may seem surprising that there is not always a Condorcet candidate, but indeed there need not be. Consider the following chart of ballots:

25	15	5	25	20	10
A	A	B	B	C	C
B	C	A	C	A	B
C	B	C	A	B	A

This table indicates that 25 voters prefer A to B to C, 15 prefer A to C to B, and so on. If we compare A to B, we see that $25 + 15 + 20 = 60$ voters prefer A to B, while $5 + 25 + 10 = 40$ prefer B to A, so A would win a two person contest against B. Comparing B to C, B wins 55 to 45, and finally C beats A 55 to 45. Thus in this contest there is no candidate who beats the other two.

Vote Tallying Methods

1. **Plurality.** In practice this usually means that the candidate who receives the most first place rankings is the winner. This can be extended to rank the candidates by the number of first place votes that they get. Though we still assume that each voter has a personal ranking of all the candidates, only the first place votes count. Consider this example, based on an actual election:

22	23	15	29	7	4
D	D	H	H	J	J
H	J	D	J	H	D
J	H	J	D	D	H

D wins this race with 45% of the vote, while H is a very close second with 44%. H and J were in fact perceived as close to each other by most voters—only 19% did not rank H and J next to each other. With J out of the race, H wins 51 to 49, and H also beats J, so H is a Condorcet winner—plurality voting does not always elect a Condorcet winner. Also, since the result changes when J drops out, plurality does not have the IIA property. This is a classic case of "splitting the vote," in which a minor candidate, who could not win over either of the other candidates, splits off enough votes from one of them to give the election to the other.

2. **Borda count.** If there are n candidates, each receives $n - 1$ points for every first place vote, $n - 2$ points for every second place vote, and so on down to zero points for a last place vote. For example, with candidates A–D, suppose the voters fill out ballots in the following way:

14	10	8	4	1
A	C	D	B	C
B	B	C	D	D
C	D	B	C	B
D	A	A	A	A

A gets 14 first place votes and 23 last place votes, so A gets $14 \cdot 3 = 42$ points. B gets 4 first place votes, 24 second place votes, 9 third place votes for a total of $4 \cdot 3 + 24 \cdot 2 + 9 \cdot 1 = 69$ points. C gets 11 first place votes, 8 second place votes, 18 third place votes for $11 \cdot 3 + 8 \cdot 2 + 18 \cdot 1 = 67$ points. D gets 8 first place votes, 5 second place votes, 10 third place votes for $8 \cdot 3 + 5 \cdot 2 + 10 \cdot 1 = 44$ points. The final ranking is thus B, C, D, A. A would win if the plurality system were used, but comes in last here. If D drops out of the race, the chart becomes:

14	10	8	4	1
A	C	C	B	C
B	B	B	C	B
C	A	A	A	A

Now A has 28 points, B has 41, and C has 42. When D drops out, C passes B, so Borda count does not have the IIA property.

3. **Instant Runoff Voting (IRV).** This method is used to elect a single winning candidate. A closely related system called Single Transferable Vote can be used to elect multiple candidates with a single set of ballots, for example, if two seats on a board of education are to be filled.

   In IRV, any candidate at any stage with more than half of the first place votes wins. If there is no such candidate, the candidate (or candidates) with the fewest first place votes is eliminated; the first place votes for the eliminated candidate(s) are transferred to the voters' next choice still in the race. The process is repeated until one candidate has more than half of the first place votes.

   Example: use the same starting ballots as for Borda Count. B is eliminated first. The ballots then look like this:

14	10	8	4	1
A	C	D	D	C
C	D	C	C	D
D	A	A	A	A

Now C has the fewest first place votes and is eliminated. The ballots are now:

14	10	8	4	1
A	D	D	D	D
D	A	A	A	A

Another example:

6	5	4	2
A	C	B	D
D	A	C	B
B	B	D	A
C	D	A	C

6	5	4	2
A	C	B	B
B	A	C	A
C	B	A	C

6	5	4	2
A	A	B	B
B	B	A	A

First D is eliminated, then C, then A wins with a majority. Now consider this:

6	5	4	2
A	C	B	D
D	A	C	A
B	B	D	B
C	D	A	C

6	5	4	2
A	C	B	A
B	A	C	B
C	B	A	C

6	5	4	2
A	C	C	A
C	A	A	C

Comparing the ballots to the previous example, A has been moved above B on 2 ballots, but by being ranked higher A goes from the winner to a loser in this contest. IRV does not have the monotonicity property.

4. **Agenda voting.** The candidates are placed in an agenda by listing them in some order. The first two candidates are compared by counting how many voters prefer one to the other; the least preferred is eliminated, and the winner is compared to the third candidate, the winner of that contest compared to the fourth candidate, and so on. The winner is the final surviving candidate.

2	5	4
A	C	B
B	A	D
D	B	C
C	D	A

   Use the agenda A, B, C, D. Then A beats B, 7–4; C beats A, 9–2; D beats C, 6–5. D is the winner, but notice that B beats D 11–0! Agenda voting does not have the Pareto property. Using the agenda A, C, D, B the final contest is between D and B and B wins—the winner depends on the agenda order, not just the ballots.

5. **Round Robin.** Every candidate is compared to every other in a sequence of head-to-head contests. Each candidate gets a point for each such contest won, and ½ point for each tie. The candidates are ranked by points.

14	10	8	4	1
A	C	D	B	C
B	B	C	D	D
C	D	B	C	B
D	A	A	A	A

   A beats nobody, so B, C, and D each get a point for beating A. B loses to C, C gets a point; B beats D, B gets a point; C beats D, C gets a point. The totals are A:0; B:2; C: 3; D:1. The final ranking is C, B, D, A. Round Robin does not have the IIA property.

Arrow's Theorem

In his doctoral thesis Kenneth Arrow proved a theorem, roughly stated, that no method for producing a ranking of preferences for society as a whole based on the preferences of individual voters is "fair". We will look at a version of this theorem. Suppose we say that a "fair" voting system is one with the monotonicity, Pareto, and IIA properties. What can we say about such a system? Does one even exist?

It turns out that there is exactly one ranking system with these three properties, but it is surely unacceptable: a dictator. That is, everyone gets to vote, but only one ballot counts. Let's check the three properties:

Monotonicity. If some voters other than the dictator move a candidate up, then there is no change in the outcome, so certainly the candidate does not move down. If the dictator moves a candidate up, that candidate goes up in the final ranking.

Pareto. If all voters prefer A to B, including the dictator, then A beats B.

IIA. If C drops out of the race, the remainder of the candidates are still ranked as they are on the dictator's ballot, so there is no change in their relative ranking.

Here's the version of Arrow's Theorem that we prove:

> The only ranking system that has the Monotonicity, Pareto, and IIA properties is the system in which the final outcome is exactly the same as a single ballot, namely, the ballot of a single voter called the dictator.

How can we prove something so general? We need somehow to be able to talk about any voting system at all, not just the ones that are known and studied. Imagine that we have a "black box," perhaps a computer, into which we feed ballots containing individual rankings of the candidates; out comes the final ranking, the result of the vote. We assume that however this computer tallies the votes, it does have the Monotonicity, Pareto, and IIA properties. We then imagine feeding carefully selected sets of ballots into the machine and we know certain things must happen because of the three properties. We gradually learn more and more about how the machine operates, and we finally can prove that it is paying attention to just one ballot, the ballot of the dictator.

Let's call the population of all voters P. If X is some set of the voters in P and x and y are two candidates, we say that X forces x over y if whenever every voter in X favors x over y then in fact x does beat y in the final ranking.

Now suppose we feed in some ballots, with every voter in X ranking x over y and every other voter ranking y over x, and suppose that in the final ranking x beats y. Does this mean that X forces x over y? In other words, does x beat y even if some of the voters not in X rank x over y? Yes, by monotonicity—if some voters move x up then x can't move down in the final ranking, so x still beats y.

It would seem that in any "reasonable" voting system, if X forces A over B then X should be able to force any candidate at all over any other candidate. But is this really true for our particular vote tallying machine? The answer is yes, but it takes a bit of work to show this.

Let's call X a *dictating set* if X forces any x over any other y. We want to show that if X forces A over B then X is a dictating set.

First, to simplify later arguments, we show that our vote machine cannot produce a tie between any candidates. Suppose that in fact some set of ballots produces a tie between A and B. By the IIA property, all that matters is how A and B are ranked on the ballots. Suppose that X is the set of all voters who rank A over B and Y is the set of all other voters, who rank B over A, and that the result has A and B tied.

Now imagine that we put in the following ballots: every ballot from X ranks A over C over B, and every ballot in Y ranks C over B over A. Since the ballots are identical to the original ballots as far as A and B are concerned, the IIA property insures that A and B are still tied. But also all ballots have C over B, so by the Pareto property, C beats B in the final ranking, and since A and B are tied, C beats A also.

Now imagine feeding another set of ballots into the machine. This time every ballot from X ranks A over B over C, and every ballot from Y ranks B over C over A. Again A and B are tied, by the IIA property, but this time B is above C on every ballot, so in the final ranking C is below A and B.

But in the previous two sets of ballots, removing B leaves exactly the same set of A versus C ballots: all X ballots are A over C and all Y ballots are C over A. This means, by the IIA property, that the final ranking of A and C is the same in both cases, yet we saw that C beat A in one and A beat C in the other. This contradiction means that it can't be true that the original set of ballots produced a tie.

Now we start to investigate dictating sets. There certainly is a dictating set: the set of all voters. If all voters rank any x over some other y, then the Pareto property guarantees that x really beats y. We want to show there is a dictating set of size 1, or in other words, there is a dictator. We make a series of statements about the forcing behavior of sets, which taken together show that if X forces A over B for any two particular candidates then X is actually a dictating set.

I. Suppose X forces A over B. Then for any candidate C, X forces A over C and C over B.

To see this, imagine that we make up ballots A over B over C for X and B over C over A for everyone else. Then the final ranking has A over B, since X forces A over B, and also B over C, by the Pareto property. So of course A is over C in the final ranking. Since all X ballots have A over C and all other ballots have C over A, this means that X forces A over C, which is half of what we want to show.

Now suppose we feed in ballots C over A over B for X and B over C over A for all other ballots. Again A beats B, and this time C beats A by the Pareto property. Hence C beats B and X forces C over B since the X voters favor C over B while all other ballots favor B over C.

II. If X forces A over B then X forces B over A.

By number I, X forces A over C. But then applying I to A and C, we know that X forces A over Z and Z over C, no matter which candidate Z is, so X forces B over C. Applying I to B and C, X forces B over Z and Z over C, no matter which candidate we put in for Z. In particular, putting A in for Z, X forces B over A.

III. If X forces A over B then X forces C over D, for any candidates C and D.

From I and II we already know that X forces A over B, B over A, A over anything, anything over A, B over anything, and anything over B. Thus X forces C over A and A over D. If we feed in ballots with C over A over D on all X ballots, and D over C on all others, then the final ranking must have C over A over D, because X forces this to happen. Removing candidate A, we have ballots on which everyone in X ranks C over D and everyone else ranks D over C, and C beats D by IIA. Thus X forces C over D.

Thus, if X forces any particular A over B, then X forces any candidate over any other, so X is a dictating set.

Now imagine that X is any dictating set, and we split X into two pieces, say V and W. Suppose we feed in these ballots:

V	*W*	others
A	C	B
B	A	C
C	B	A

Since X dictates we get A over B in the final ranking, so the final ranking must be C–A–B, A–B–C, or A–C–B.

If the final ranking is C–A–B then W forces C over B, so W is a dictating set.

If the final ranking is A–B–C then V forces A over C, so V is a dictating set.

If the final ranking is A–C–B then V forces A over C and W forces C over B, so both V and W are dictating sets, but that's impossible, so in fact this can't happen.

The upshot is that if we split a dictating set in two, then one of the pieces is still a dictating set. If we start with the dictating set consisting of all voters we can split it into two, and find a smaller dictating set. Then we can split that one, and get a smaller dictating set. If we continue this, we eventually have to get a dictating set containing just one voter—a dictator! Thus we have shown that our ballot counting machine must be looking at just one ballot to compute the final ranking.

There is one remaining question: if there is a dictator, does the voting system really have all three properties? It does:

Monotonicity: Suppose A beats B—this is because the dictator ranks A over B. Now if one or more ballots is changed to move A up, A still beats B, because the dictator's ballot still ranks A over B.

Pareto: If every ballot ranks A over B, then the dictator's does, and so A really does beat B.

IIA: Suppose A beats B–this is because the dictator ranks A over B. Suppose candidate C drops out. The dictator still has A over B, so A still beats B.

So this most unreasonable of voting systems, a dictator, has some very reasonable properties!

Approval Voting

There is no perfect ranking system. We can therefore pick the least bad ranking system or investigate changing the rules a bit. One fairly recent such voting system, with many fans, has been adopted by a variety of organizations, but not by any governmental bodies in the US. In this system, individual voters can vote for as many candidates as they wish, but they cannot distinguish their level of support or preference. The intent is that a voter will divide the candidates into just two groups, the "acceptable" and the "unacceptable", and vote for all the acceptable ones. The candidate with the largest total wins.

Since AV is not a ranking system, some of the properties we looked at do not apply directly to AV. Still, it is useful to consider what can be said.

Condorcet. AV does not have the Condorcet property.

33	33	34
A	$\underline{\text{B}}$	$\underline{\text{C}}$
$\underline{\text{B}}$	A	A
C	C	B

Here A gets 33 votes, B gets 66, C gets 34, so B wins, but A is a Condorcet winner. (In a sense A is a very weak Condorcet winner—A beats B and C on the strength of those voters who rank A as unacceptable.)

Monotonicity. If A moves up on some ballots, A can't be hurt. If A starts below the approval line and stays below, or starts above it and stays above, then there is no change. If A moves from below the line to above it, then A can't do worse. Thus, AV does have the monotonicity property.

Pareto. If everyone prefers A to B, then A gets at least as many votes as B, since whenever B is above the line then A is also. It is possible that A and B could be tied. Thus, AV satisfies a "weak Pareto property:" If everyone prefers A to B then A can't lose to B, but A might not beat B. If A and B were tied for first place the tie would have to be broken somehow, in which case A might lose to B, depending on the method used to break the tie.

IIA. If a candidate drops out and the approval lines don't change (that is, no remaining candidate moves from below the line to above the line), then the relative ranking of the remaining candidates doesn't change, so in this interpretation AV does have the IIA property.

Steven Brams is a political scientist who has studied Approval Voting (and incidentally has studied the application of game theory to politics). He wrote a book with Peter Fishburn on the subject and the following article which can be found on the web.

Approval Voting and the Good Society
Steven J. Brams

Proposed independently by several analysts in the 1970s, *approval voting* (AV) is a voting procedure in which voters can vote for, or approve of, as many candidates as they wish in multicandidate elections—that is, elections with more than two candidates. Each candidate approved of receives one vote, and the candidate with the most votes wins. In the United States, the case for AV seems particularly strong in primary and nonpartisan elections, which often draw large fields of candidates.

AV has several compelling advantages over other voting procedures:

1. *It gives voters more flexible options*. They can do everything they can under plurality voting (PV)—vote for a single favorite—but if they have no strong preference for one candidate, they can express this fact by voting for all candidates they find acceptable. In addition, if a voter's most preferred candidate has little chance of winning, that voter can vote for both a first choice *and* a more viable candidate without worrying about wasting his or her vote on the less popular candidate.

2. *It helps elect the strongest candidate*. Today the candidate supported by the largest minority often wins, or at least makes the runoff if there is one. Under AV, by contrast, the candidate with the greatest overall support will generally win. In particular, 'Condorcet candidates,' who can defeat every other candidate in separate pairwise contests, almost invariably win under AV, whereas under PV they often lose because they split the vote with one or more other centrist candidates.

3. *It will reduce negative campaigning*. AV induces candidates to try to mirror the views of a majority of voters, not just cater to minorities whose voters could give them a slight edge in a crowded plurality contest. It is thus likely to cut down on negative campaigning, because candidates will have an incentive to try to broaden their appeals by reaching out for approval to voters who might have a different first choice. Lambasting such a choice would risk alienating this candidate's supporters and losing their approval.

4. *It will increase voter turnout*. By being better able to express their preferences, voters are more likely to vote in the first place. Voters who think they might be wasting their votes, or who cannot decide which of several candidates best represents their views, will not have to despair about making a choice. By not being forced to make a single—perhaps arbitrary—choice, they will feel that the election system allows them to be more honest, which will make voting more meaningful and encourage greater participation in elections.

5. *It will give minority candidates their proper due*. Minority candidates will not suffer under AV: their supporters will not be torn away simply because there is another candidate who, though less appealing to them, is generally considered a stronger contender. Because AV allows these supporters to vote for *both* candidates, they will not be tempted to desert the one who is weak in the polls, as under PV. Hence, minority candidates will receive their true level of support under AV, even if they cannot win. This will make election returns a better reflection of the overall acceptability of candidates, relatively undistorted by strategic voting, which is important information often denied to voters today.

6. *It is eminently practicable*. Unlike more complicated ranking systems, which suffer from a variety of theoretical as well as practical defects, AV is simple for voters to understand and use. Although more votes must be tallied under AV than under PV, AV can readily be implemented on existing voting machines. Because AV does not violate any state constitutions in the United States (or, for that matter, the constitutions of most countries in the world), it requires only an ordinary statute to enact.

Probably the best-known official elected by AV today is the secretary-general of the United Nations. AV has been used in internal elections by the political parties in some states, such as Pennsylvania. Bills to implement AV have been introduced in several state legislatures. In 1987, a bill to enact AV in certain statewide elections passed the Senate but not the House in North Dakota. In 1990, Oregon used AV in a statewide advisory referendum on school financing, which presented voters with five different options and allowed them to vote for as many as they wished.

In 1987 and 1988, several scientific and engineering societies inaugurated the use of AV. It has worked well in finding consensus candidates, and all the societies continue to use it today. These societies are:

- The Mathematical Association of America (MAA), with about 32,000 members;
- The Institute of Management Science (TIMS), with about 7,000 members;
- The American Statistical Association (ASA), with about 15,000 members;
- The Institute of Electrical and Electronics Engineers (IEEE), with about 320,000 members.

In addition, the Econometric Society has used AV (with certain emendations) to elect fellows since 1980; likewise, since 1981 the selection of members of the National Academy of Sciences at the final stage of balloting has been based on AV. Coupled with many colleges and universities that now use AV—from the departmental level to the school-wide level—it is no exaggeration to say that several hundred thousand individuals have had direct experience with AV.

Beginning in 1987, AV was used in some competitive elections in countries in Eastern Europe and the Soviet Union. It continues to be used there today, where it is usually 'disapproval voting' because voters are only permitted to cross off names on ballots. But this procedure is logically equivalent to AV: candidates not crossed off are, in effect, approved of, although psychologically there is almost surely a difference between approving and disapproving of candidates.

As cherished a principle as 'one person, one vote' is in single-winner elections, such as for president, it is probably an anachronism today. Western democracies, as well as developed and developing countries in other parts of the world, could benefit more from the alternative principle of 'one candidate, one vote,' whereby voters are able to make a judgment about whether each candidate on the ballot is acceptable or not.

The latter principle makes the tie-in of a vote not to the voter but rather to the candidates, which is surely more egalitarian than artificially restricting voters to casting only one vote in multicandidate races. This principle also affords voters an opportunity to express their intensities of preference by approving, for example, of all except the one candidate they may despise.

More than intellectual issues are at stake here. With some 500,000 elected officials serving in approximately 80,000 governments in the United States alone, and probably similar proportions in other countries, the consequences of using different election procedures to implement democratic elections are great. Procedures are not innocuous. They can make a difference not only on who is elected but also on what ultimately becomes public policy.

AV is a strikingly simple election reform for finding consensus choices in single-winner elections. On the other hand, in elections with more than one winner, such as for a council or legislature, AV would not be desirable if the goal is to mirror a diversity of views, especially of minorities. Minorities, nonetheless, will derive indirect benefit from AV, because their candidates will retain their own supporters; at the same time, more mainstream candidates will be forced to reach out to these supporters for the approval *they* need to win.

I believe a good society must be majoritarian in its choice of leaders—at least those selected in single-winner elections. At the same time, it must make these leaders responsive to minority views. AV is an election reform that deftly accomplishes both ends in a practicable way.

Steven J. Brams is professor of politics at New York University. He is co-author, with Peter C. Fishburn, of *Approval Voting* (1983) and author of *Theory of Moves* (1993). This article is available at `http://bcn.boulder.co.us/government/approvalvote/goodsoc.html`