

Calculus I  
Answers to Sample Exam 1

1. Find  $\lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x - 4}$  or explain why it doesn't exist.

**Answer.**  $\lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x - 4} \frac{\sqrt{x} + 2}{\sqrt{x} + 2} = \lim_{x \rightarrow 4} \frac{x - 4}{(x - 4)(\sqrt{x} + 2)} = \lim_{x \rightarrow 4} \frac{1}{(\sqrt{x} + 2)} = 1/4.$

2. Compute the derivative of  $f(x) = \sqrt{x+1}$  directly from the limit definition of the derivative. **Do not** use any shortcuts.

**Answer.**

$$\begin{aligned} f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{\sqrt{x + \Delta x + 1} - \sqrt{x + 1}}{\Delta x} \frac{\sqrt{x + \Delta x + 1} + \sqrt{x + 1}}{\sqrt{x + \Delta x + 1} + \sqrt{x + 1}} \\ &= \lim_{\Delta x \rightarrow 0} \frac{x + \Delta x + 1 - (x + 1)}{\Delta x (\sqrt{x + \Delta x + 1} + \sqrt{x + 1})} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\Delta x}{\Delta x (\sqrt{x + \Delta x + 1} + \sqrt{x + 1})} \\ &= \lim_{\Delta x \rightarrow 0} \frac{1}{\sqrt{x + \Delta x + 1} + \sqrt{x + 1}} \\ &= \frac{1}{2\sqrt{x + 1}} \end{aligned}$$

3. Compute  $\frac{d}{dt}(t^8 + 6t^7 - 18t^2 + 2t - 1/t).$

**Answer.**  $8t^7 + 42t^6 - 36t + 2 + 1/t^2$

4. Compute  $\frac{d}{dx} x^3 \sqrt{x^3 + 3x^2 + 5}.$

**Answer.**  $x^3(1/2)(x^3 + 3x^2 + 5)^{-1/2}(3x^2 + 6x) + 3x^2 \sqrt{x^3 + 3x^2 + 5}$

5. Compute  $\frac{d}{dx} \frac{1}{(x^2 + 3)^2}.$

**Answer.**  $-2(x^2 + 3)^{-3}(2x)$

6. Compute  $\frac{d}{dx} \frac{5(x^2 + 3)}{\sqrt{15 + x^2}}$ .

**Answer.**  $5 \cdot \frac{2x\sqrt{15 + x^2} - (x^2 + 3)(1/2)(15 + x^2)^{-1/2}(2x)}{15 + x^2}$

7. Compute  $\frac{d}{dx} (\sqrt{x} + x^2)^{47} (x^2 + 2)$ .

**Answer.**  $47(\sqrt{x} + x^2)^{46} (\frac{1}{2}x^{-1/2} + 2x)(x^2 + 2) + (\sqrt{x} + x^2)^{47} 2x$

8. Compute  $\frac{d}{dx} \sqrt{x^2 + \sqrt{x^3 - x}}$ .

**Answer.**  $\frac{1}{2} \left( x^2 + \sqrt{x^3 - x} \right)^{-1/2} \left( 2x + \frac{1}{2} (x^3 - x)^{-1/2} (3x^2 - 1) \right)$

9. Suppose a ball is thrown straight up so that its height at time  $t$  is  $30t - 16t^2$  meters. How fast is the object going at time  $t = 1$ ?

**Answer.** We want to know the derivative of the function  $30t - 16t^2$  at  $t = 1$ . The derivative is  $30 - 32t$  so at  $t = 1$  the ball is going  $-2$  meters per second.

10. Find an equation for the tangent line to  $f(x) = 4x^2$  at  $x = 1$ .

**Answer.** The derivative is  $8x$ , so when  $x = 1$  the slope of the tangent line is 8. Since  $f(1) = 4$ , the tangent line is  $y - 4 = 8(x - 1)$  or  $y = 8x - 4$ .