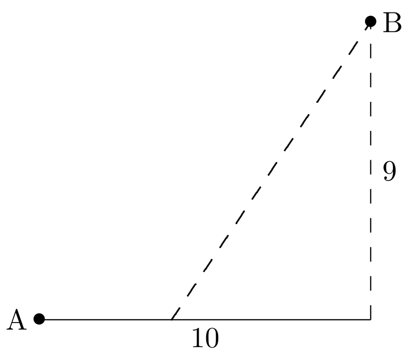
Calculus I
Sample Exam 3

Do all ten problems. For maximum credit, show your work and justify your answers; answers alone will seldom receive full credit. If you show your work and if your answer is wrong, you may still receive partial credit. You need not simplify answers. Each of the ten problems is worth 10 points.

1. Find the global maximum and minimum points ($x$ and $y$ coordinates) of $x - \cos x$ on $[-\pi, \pi]$.

2. A cylinder is to be manufactured to hold 1 liter (1000 $\text{cm}^3$). Find the radius and height of the cylinder that has the smallest possible surface area (top plus bottom plus side).

3. One town, labeled A, is located on a highway. Another, B, is located 9 miles north of a point 10 miles east of A. The speed limit on the highway is 50 mph, and a new road is to be built from the highway to B, as indicated. The speed limit on the new road will be 25 mph. Where should the new road join the highway to minimize the (legal) travel time from A to B?




4. A box with a bottom but no top is to be made from a piece of cardboard 1 foot by 2 feet, by cutting a square out of each corner. Find the shape of the box with largest volume.

5. An airplane is flying at a constant altitude of 5 km, on a path that will take it directly over a radar dish. When the radar reports that the plane is 10 km away (measured on a straight line between plane and radar) it also reports that the distance between radar and plane is decreasing at 600 km/hr. What is the actual (that is, horizontal) speed of the plane?

6. The light from a rotating beacon shines on a straight wall 8 feet from the beacon. The beacon makes two revolutions per minute. How fast is the spot of light on the wall moving when the spot is 10 feet from the beacon (measured in a straight line between the beacon and the spot)?

7. A man 6 ft tall walks at the rate of 3 ft/sec away from a streetlight that is 10 ft above the ground. At what rate is his shadow lengthening?

8. Water is poured into a conical container at the rate of 2 $\text{m}^3$/sec. The cone points directly down, and it has a height of 10 m and a base radius of 5 m. How fast is the water level rising when the water is 3 m deep (at its deepest point)?

9. Using $x_0 = 3$ as the initial guess, compute the $x_1$ approximation to $\sqrt{11}$ by Newton's method.

10. The side of a square is measured at 2 kilometers, but the method of measurement is accurate to only one meter. Use differentials to estimate the largest and smallest area that the square might contain.