Calculus II
Answers to Sample Exam 1

1. Write a sum of 5 terms to approximate the integral $\int_1^2 \frac{1}{x}\, dx$. DO NOT simplify or combine terms; leave your answer as a sum of 5 quantities.

   Answer. $\frac{1}{5} \cdot 1 + \frac{1}{5} \cdot \frac{5}{6} + \frac{1}{5} \cdot \frac{5}{7} + \frac{1}{5} \cdot \frac{5}{8} + \frac{1}{5} \cdot \frac{5}{9}$

2. Find the derivative of $f(x) = \int_{x^2}^{2} \frac{\tan^3 t}{t}\, dt$.

   Answer. Rewrite the integral as $f(x) = -\int_{2}^{x^2} \frac{\tan^3 t}{t}\, dt$. Then using the Fundamental Theorem of Calculus and the chain rule, the derivative is

   $$-\frac{\tan^3 x^2}{x^2}(2x) = -2\frac{\tan^3 x^2}{x}.$$

3. Find $\int \frac{\ln x}{x}\, dx$.

   Answer. Use $u = \ln x$, so $\int \frac{\ln x}{x}\, dx = \int u\, du = u^2/2 + C = \ln(x)^2/2 + C$

4. Find $\int \frac{16}{\sqrt{16 - x^2}}\, dx$.

   Answer. Use $x = 4 \sin u$. Then

   $$\int \frac{16}{\sqrt{16 - x^2}}\, dx = 16 \int \frac{1}{\sqrt{16 - 16 \sin^2 u}} 4 \cos u\, du$$

   $$= 16 \int 1\, du = 16u + C = 16 \arcsin(x/4) + C$$

5. Compute $\int_0^{\pi/2} \cos^2 x\, dx$.

   Answer. $\int_0^{\pi/2} \cos^2 x\, dx = \frac{1}{2} \int_0^{\pi/2} 1 + \cos(2x)\, dx = \frac{x}{2} + \frac{\sin(2x)}{4} \Big|_0^{\pi/2} = \frac{\pi}{4}$

6. Find $\int (x^2 + 2x - 1)e^x \, dx$.

   Answer. Use integration by parts twice.

   $$\int (x^2 + 2x - 1)e^x \, dx = e^x(x^2 + 2x - 1) - \int (2x + 2)e^x \, dx$$
   $$= e^x(x^2 + 2x - 1) - (e^x(2x + 2) - 2e^x) + C$$
   $$= (x^2 - 1)e^x + C$$

7. Find $\int \tan x \sec^3 x \, dx$.

   Answer. Use $u = \sec x$. $\int \tan x \sec^3 x \, dx = \int u^2 \, du = u^3/3 + C = \dfrac{\sec^3 x}{3} + C$

8. Find $\int \dfrac{1 + \sin x}{1 - \sin^2 x} \, dx$.

   Answer.

   $$\int \frac{1 + \sin x}{1 - \sin^2 x} \, dx = \int \frac{1 + \sin x}{\cos^2 x} \, dx = \int \sec^2 x \, dx + \int \sec x \tan x \, dx$$
   $$= \tan x + \sec x + C$$

9. Find $\int \dfrac{-1}{(4 + x^2)^{(3/2)}} \, dx$.

   Answer. Use $x = 2 \tan u$.

   $$\int \frac{-1}{(4 + x^2)^{(3/2)}} \, dx = -\frac{1}{4} \int \cos u \, du = -\frac{1}{4} \sin u + C$$
   $$= -\frac{1}{4} \sin \arctan(x/2) + C = \frac{-x}{4\sqrt{4 + x^2}} + C$$

10. Find $\int \dfrac{4x - 2}{(x - 1)(x + 2)} \, dx$.

    Answer.

    $$\int \frac{4x - 2}{(x - 1)(x + 2)} \, dx = \int \frac{2}{3} \frac{1}{x - 1} + \frac{10}{3} \frac{1}{x + 2} \, dx = \frac{2}{3} \ln |x - 1| + \frac{10}{3} \ln |x + 2| + C$$