1. Find the area bounded by the curves \( y = 6x - x^2 \) and \( y = x^2 - 2x \).

**Answer.** The two curves intersect when \( 6x - x^2 = x^2 - 2x \), or \( 0 = 2x^2 - 8x \), with solutions \( x = 0 \) and \( x = 4 \). A quick sketch shows that \( y = 6x - x^2 \) is the larger between 0 and 4. Then the area is 
\[
\int_0^4 (6x - x^2) - (x^2 - 2x) \, dx = \int_0^4 8x - 2x^2 \, dx = \frac{64}{3}.
\]

2. An object moves so that its acceleration at time \( t \) is given by \( a(t) = -5 \). Its initial position is \( s(0) = 10 \) and its initial velocity is \( v(0) = 2 \). Find \( s(t) \) and \( v(t) \).

**Answer.**
\[
v(t) = v(0) + \int_0^t a(u) \, du = 2 - 5t; \quad s(t) = s(0) + \int_0^t v(u) \, du = 10 + 2t - \frac{5t^2}{2}.
\]

3. Find the volume generated when the area between \( y = 2x^2 \) and \( y = 8 \) from \( x = 0 \) to \( x = 2 \) is rotated around the line \( y = 10 \).

**Answer.**
\[
\pi \int_0^2 (10 - 2x^2)^2 - 2^2 \, dx = \pi \int_0^2 4x^4 - 40x^2 + 96 \, dx = \pi \left( \frac{4}{5} \cdot 32 - \frac{40}{3} \cdot 8 + 192 \right) = \frac{1664\pi}{15}.
\]

4. An object moves so that its velocity is \( v(t) = \sin(t) \). Find its average velocity between \( t = \pi/2 \) and \( t = 2\pi \).

**Answer.**
\[
\frac{1}{2\pi - \pi/2} \int_{\pi/2}^{2\pi} \sin t \, dt = \frac{2}{3\pi} \left( -\cos t \right) \bigg|_{\pi/2}^{2\pi} = -\frac{2}{3\pi}.
\]

5. A rope 100 meters long is hanging straight down from the top of a building. The density of the rope is 2 kg per meter. Find the amount of work required to lift the entire rope to the top of the building. Use 9.8 for the acceleration due to gravity.

**Answer.**
\[
\int_0^{100} 2 \cdot 9.8(100 - y) \, dy = 98000 \text{ N-m}.
\]

6. A spring has a natural length of 1/2 meter. It takes a force of 5 newtons to stretch the spring to 1 meter in length. Find the amount of work required to compress the spring from its natural length to 1/4 meter.

**Answer.** By Hooke’s Law, \( k(1/2) = 5 \) so \( k = 10 \). Then the work is 
\[
\int_0^{1/4} 10x \, dx = \frac{5}{16},
\]
or \[
\int_{1/2}^{1/4} 10 \left( x - \frac{1}{2} \right) \, dx = \frac{5}{16}.
\]
7. A beam 4 meters long has density \( \sigma(x) = 2 + \sin(\pi x) \), where \( x \) is the distance from the left end of the beam. Find the center of mass.

**Answer.** 
\[
M = \int_0^4 2 + \sin(\pi x) \, dx = 8 
\]

\[
M_0 = \int_0^4 x(2 + \sin(\pi x)) \, dx
\]
\[
= \int_0^4 2x \, dx + \int_0^4 x \sin(\pi x) \, dx
\]
\[
= 16 - \frac{x \cos(\pi x)}{\pi} \bigg|_0^4 + \int_0^4 \frac{\cos(\pi x)}{\pi} \, dx
\]
\[
= 16 - \frac{4}{\pi} + 0
\]
\[
= \frac{4(4\pi - 1)}{\pi}
\]

using integration by parts. Then \( \bar{x} = \frac{M_0}{M} = \frac{4\pi - 1}{2\pi} \approx 1.84 \)

8. Compute \( \int_1^\infty x^{-3/2} \, dx \).

**Answer.** 
\[
\int_1^\infty x^{-3/2} \, dx = \lim_{D \to \infty} -2x^{-1/2} \bigg|_1^D = \lim_{D \to \infty} -2D^{-1/2} + 2 = 2.
\]

9. Set up an integral to compute the length of the curve given by \( f(x) = \sin(x^2) \) for \( x \) in \([0, \pi]\). Do not evaluate the integral.

**Answer.** 
\[
\int_0^\pi \sqrt{1 + 4x^2 \cos^2(x^2)} \, dx
\]

10. Set up an integral to compute the surface area generated when \( f(x) = 2^x \), for \( x \) in \([0, 2]\), is rotated around the \( x \)-axis. Do not evaluate the integral.

**Answer.** 
\[
\int_0^2 2\pi 2^x \sqrt{1 + \ln(x)^2 2^{2x}} \, dx
\]