Calculus III—Math 225

SAMPLE EXAM 1

When asked to provide a vector as an answer, write it in the form $\langle a, b, c\rangle$.

1. Let $\mathbf{a} = \langle 1, 0, -1\rangle$, $\mathbf{b} = \langle -2, 3, 5\rangle$. Find $|\mathbf{a}|$, $\mathbf{a} - \mathbf{b}$, and a unit vector in the same direction as $\mathbf{a} - \mathbf{b}$.
2. Let $\mathbf{v} = \langle 5, -1, 6\rangle$, $\mathbf{w} = \langle -2, 2, -4\rangle$. Find the cosine of the angle between $\mathbf{v}$ and $\mathbf{w}$.
3. Find a vector perpendicular to both $\langle 1, 2, 2\rangle$ and $\langle -3, 1, 5\rangle$.
4. Find the vector projection of $\mathbf{a}$ onto $\mathbf{b}$, using $\mathbf{a} = \langle 2, 2, 2\rangle$ and $\mathbf{b} = \langle 1, -1, -1\rangle$.
5. Find a vector function for the line that is the intersection of the planes $x + y + z = 2$ and $3x - 2y - z = -5$.
6. Find an equation for the plane that is perpendicular to both of the planes $x+y+z = 2$ and $3x - 2y - z = -5$ and contains the point $(1, 1, 1)$.
7. Using $\mathbf{r}(t) = \langle t^2 + 2, t^2 - 4t, 2t\rangle$, find an equation for a plane perpendicular to $\mathbf{r}$ at $(6, -4, 4)$.
8. Find the curvature of $\mathbf{r}(t) = \langle t^2+2, t^2-4t, 2t\rangle$ from the previous problem as a function of t and also find the curvature at $(6, -4, 4)$.
9. Suppose the position of an object is given by is $\mathbf{r}(t) = \langle t^2 + 2, t^2 - 4t, 2t\rangle$. Find the scalar components of acceleration, $a_{\mathbf{T}}$ and $a_{\mathbf{N}}$.
10. Suppose an object moves so that its acceleration vector is $\langle t, t^2, \sin t\rangle$, and at $t = 0$ it is at the point $(1, 1, 1)$ with velocity $\langle -2, 1, 2\rangle$. Find the vector functions $\mathbf{v}(t)$ and $\mathbf{r}(t)$.