Calculus III—Math 225

# Chapter 16 Sample Exam

1. Compute $\int_C xy\,ds$, where $C$ is given by $\langle 2\sin\theta, 2\cos\theta\rangle$, $0 \le \theta \le \pi/2$.

2. Explain how you can tell that $\mathbf{F} = \langle 3x^2\cos y, -x^3\sin y\rangle$ is conservative. Compute $\int_C \mathbf{F}\cdot d\mathbf{r}$, where $C$ is $\langle \cos t, t^2\rangle$, $0 \le t \le 1$.

3. Compute $\int_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F} = \langle x^2y^2, 3x + xy\rangle$ and $C$ is the square $(0,0) \to (1,0) \to (1,1) \to (0,1) \to (0,0)$.

4. Convert $\oint_C \mathbf{F}\cdot\mathbf{N}\,ds$, to a double integral that is ready to evaluate, including the limits, but do not evaluate the integral. The curve $C$ is the circle $x^2 + y^2 = 1$ and $\mathbf{F} = \langle ax^2, by^2\rangle$.

5. Compute $\nabla\times\mathbf{F}$, $\mathbf{F} = \langle x\cos z, y\cos z, \sin z\rangle$. Is $\mathbf{F}$ conservative? Why or why not?

6. Compute $\nabla\cdot\mathbf{F}$, $\mathbf{F} = \langle x\cos z, y\cos z, \sin z\rangle$.

7. Set up a double integral for the surface area of $\mathbf{r} = \langle u^2, u^2 - v, v^3\rangle$, $0 \le u \le 1$, $0 \le v \le 1$.

8. Compute $\iint_D \mathbf{F}\cdot\mathbf{N}\,dS$, where $\mathbf{F} = \langle y, z, x\rangle$ and $D$ is the surface $z = x^2 + y^2$ above the interior of the square with corners $(0,0)$, $(1,0)$, $(1,1)$, $(0,1)$, oriented up.

9. Compute $\int_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F} = \langle z^2, y, x\rangle$ and $C$ is the triangle $(1,0,0) \to (0,1,0) \to (0,0,1) \to (1,0,0)$.

10. Compute $\iint_D \mathbf{F}\cdot\mathbf{N}\,dS$, where $\mathbf{F} = \langle x^2z, z^2y, y^2x\rangle$ and $D$ is the surface of the cube with corners $(0,0,0)$, $(1,0,0)$, $(1,0,1)$, $(0,0,1)$, $(0,1,0)$, $(1,1,0)$, $(1,1,1)$, $(0,1,1)$, oriented outward.