Vector fields, section 16.1

Vector calculus : involves vector field

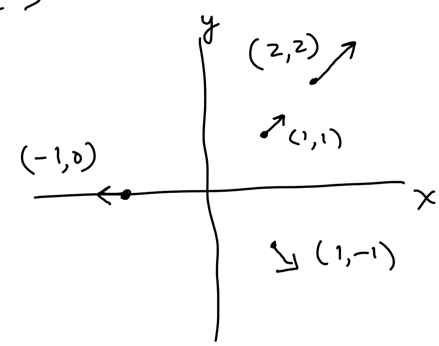
A vector field is a function that assigns a vector to every point in space.

$$f(x,y) = \langle x^2, y^3 \rangle$$

$$f(x,y,z) = \langle xz, y^2, zy^2 \rangle$$

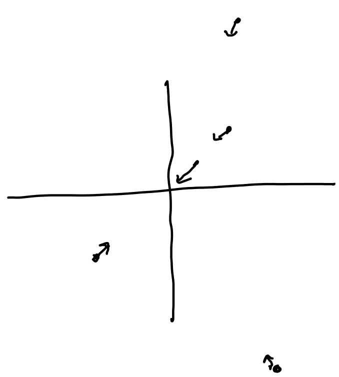
Draw the vector at the point:

$$f(x,y) = \langle x, y \rangle$$



Vector fields: force, velocities

Gravity



$$\nabla f = \langle f_x, f_y \rangle \quad \text{or} \quad \nabla f = \langle f_x, f_y, f_z \rangle$$

$$\vec{F} = \left\langle \frac{-x}{(x^2+y^2+z^2)^{3/2}}, \frac{-y}{(x^2+y^2+z^2)^{3/2}}, \frac{-z}{(x^2+y^2+z^2)^{3/2}} \right\rangle$$

parallel to $\langle -x, -y, -z \rangle$

$$|\vec{F}| = \left(\frac{x^2}{(x^2+y^2+z^2)^3} + \frac{y^2}{(x^2+y^2+z^2)^3} + \frac{z^2}{(x^2+y^2+z^2)^3} \right)^{1/2} = \left(\frac{x^2+y^2+z^2}{(x^2+y^2+z^2)^3} \right)^{1/2}$$

$$= \left(\frac{1}{(x^2+y^2+z^2)^2} \right)^{1/2} = \frac{1}{x^2+y^2+z^2}$$

1 over the square of the distance to the origin. "Inverse square law."