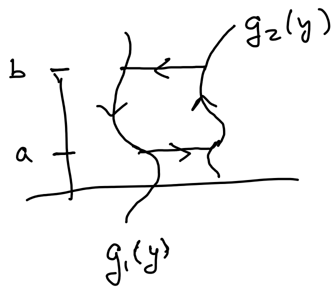
Green's Theorem, part 2, section 16.4

$$\iint_D Q_x - P_y \, dA = \underbrace{\int_{\partial D} P\,dx}_{} + \underbrace{\int_{\partial D} Q\,dy}_{}$$

$$\parallel$$

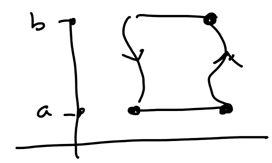
$$\underbrace{\iint_D Q_x \, dA}_{} - \underbrace{\iint_D P_y \, dA}_{}$$



$$\iint_D Q_x \, dA = \int_a^b \int_{g_1(y)}^{g_2(y)} Q_x \, dx \, dy$$

$$= \int_a^b Q(x,y) \Big|_{g_1(y)}^{g_2(y)} dy = \int_a^b \underbrace{Q(g_2(y), y)}_{} \underbrace{- Q(g_1(y), y) \, dy}_{}$$

$$\int_{\partial D} Q\,dy$$



Boundary: $y = a$; $\underbrace{x = g_2(y)}_{}$; $y = b$; $x = g_1(y)$

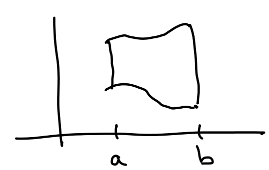
1) $\displaystyle\int_a^a Q(x,a)\,dy = 0$

2) $\displaystyle\int_a^b \underbrace{Q(g_2(y), y)}_{} \, dy$

3) $\displaystyle\int_b^b Q(x,b)\,dy = 0$

4) $\displaystyle\int_b^a Q(g_1(y), y)\,dy = \underbrace{-\int_a^b Q(g_1(y), y)\,dy}_{}$

For $\displaystyle -\iint_D P_y \, dA = \int_{\partial D} P\,dx$ need D to look like:



Examples: