Vector functions for surfaces, section 16.6

Surfaces: $z = f(x,y)$ or $f(x,y,z) = 0$ $\qquad x^2+y^2+z^2-4=0$

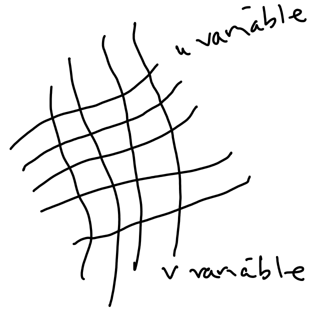
$\bar{r}(t) = \langle x(t), y(t), z(t) \rangle$

$\bar{r}(u,v) = \langle x(u,v), y(u,v), z(u,v) \rangle$

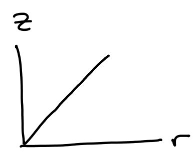
Fix v: $\bar{r}(u,v)$ is a curve as u varies.

Change v a little: new curve, a close, almost parallel, curve.

Fix u: more curves.



$\langle v\cos u, v\sin u, v \rangle$

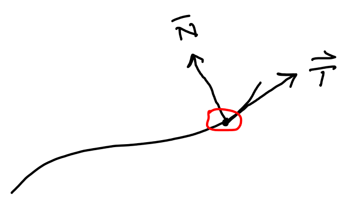


Fix v: $\langle v\cos u, v\sin u, u \rangle$ $\qquad v=1$: $\langle \cos u, \sin u, u \rangle$ helix

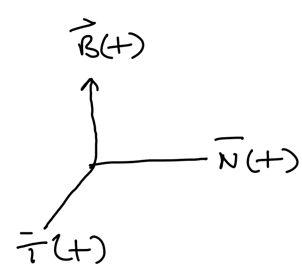
$\langle v\cos u, v\sin u, u \rangle$ helix, radius v

Fix u: $\langle v\,\underline{\cos u}, v\,\underline{\sin u}, u \rangle$

$\bar{T} = \dfrac{\bar{r}'}{|\bar{r}'|}$, $\bar{N} = \dfrac{T'}{|T'|}$



$\bar{B} = \vec{T} \times \bar{N}$ "binormal"



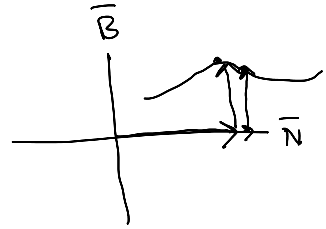
$\bar{B} - \bar{N}$ plane: perpendicular to the curve.

At each point along the curve, we can draw a curve in this $\bar{B}-\bar{N}$ plane, for example, a circle.

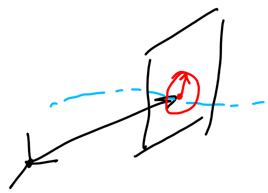
$\bar{r}(t)$.

A curve in the $\bar{B}$ - $\bar{N}$ plane looks like $c(v)\,\bar{N} + d(v)\,\vec{B}$

or $\underline{c(t,v)}\,\bar{N} + \underline{d(t,v)}\,\vec{B}$



$\bar{r}(t) + \underline{c(t,v)\,\bar{N} + d(t,v)\,\vec{B}}$



$$\bar{r}(t) + \underbrace{\frac{3}{4}\cos(v)}\,\bar{N} + \underbrace{\frac{3}{4}\sin(v)}\,\bar{B}$$

$$\bar{r}(t) + \cos(t)\,\frac{3}{4}\cos(v)\,\bar{N} + \cos(t)\,\frac{3}{4}\sin(v)\,\bar{B}$$

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cone: $\langle v\cos u, v\sin u, v\rangle = \langle 0,0,v\rangle + \langle v\cos u, v\sin u, 0\rangle$

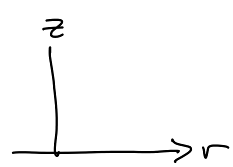
$z = f(x,y)$ (surface)

$\bar{r}(u,v) = \langle u, v, f(u,v)\rangle$ $\quad \langle x, y, f(x,y)\rangle = \bar{r}(x,y)$.

$x^2+y^2+z^2 = 1$, sphere of radius 1.

<u>Meridians:</u> $\cos(v)\,(\qquad) + \sin(v)\,\bar{k}$

vector in some fixed direction

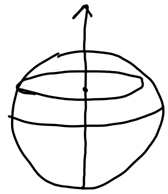


$\cos(u)\,\vec{i} + \sin(u)\,\vec{j}$

$\langle \cos(v)\cos u, \cos v \sin u, \sin(v)\rangle$

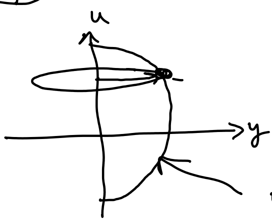
$v: 0 \to 2\pi \qquad u: 0 \to \pi$

Latitude lines:



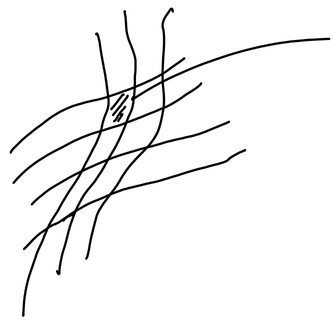
$$\langle 0,0,u\rangle + \langle \sqrt{1-u^2}\cos v, \sqrt{1-u^2}\sin v, 0\rangle$$

$$= \langle \sqrt{1-u^2}\cos v, \sqrt{1-u^2}\sin v, u\rangle$$

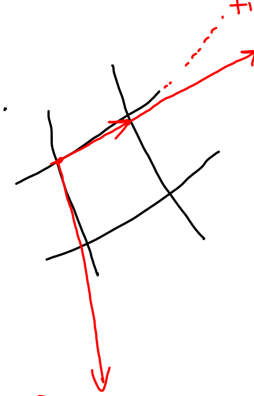


$u^2 + y^2 = 1 \qquad y = \sqrt{1-u^2}$

$\bar{r}(u,v)$



$\approx$ parallelograms.



fixed v

$\frac{\partial}{\partial u}\bar{r}(u,v)\,\Delta u$

$= \bar{r}_u(u,v)\,\Delta u$

$\frac{\partial}{\partial v}\bar{r}(u,v)\,\Delta v = \bar{r}_v(u,v)\,\Delta v$

$$\iint |\bar{r}_u \times \bar{r}_v|\, du\, dv$$