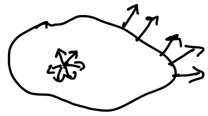
The divergence theorem, section 16.9

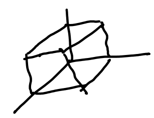
$$\int_{\partial D} \bar{F}\cdot\bar{N}\,ds = \iint_{D} \nabla\cdot\bar{F}\,dA$$



$$\iint_{\partial D} \bar{F}\cdot\bar{N}\,dS = \iiint_{D} \nabla\cdot\bar{F}\,dV$$

$\bar{N}$ is a unit vector normal to the surface, oriented outward.

<u>Example</u> $F = \langle 2x, 3y, z^2\rangle$ $D = 1\times1\times1$ cube:



<u>Top</u> $z=1.$ $r(u,v) = \langle u, v, 1\rangle$ $0 \le u \le 1,\ 0 \le v \le 1$

$r_u = \langle 1, 0, 0\rangle$

$r_v = \langle 0, 1, 0\rangle$

$r_u \times r_v = \langle 0, 0, 1\rangle$

$$\int_0^1\int_0^1 \langle 2u, 3v, 1\rangle\cdot\langle 0,0,1\rangle\,du\,dv$$

$$= \int_0^1\int_0^1 1\,du\,dv = 1$$

<u>Total</u> : $1+2+3+0+0+0$

$$\int_0^1\int_0^1\int_0^1 5+2z\,dz\,dy\,dx = \int_0^1\int_0^1 5z+z^2\Big|_0^1 dy\,dx = \int_0^1\int_0^1 6\,dy\,dx = 6\underbrace{\int_0^1\int_0^1 dy\,dx}_{1}$$

$$= 6$$

$$\nabla\cdot\langle 2x, 3y, z^2\rangle = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right\rangle\cdot\langle 2x, 3y, z^2\rangle$$

$$= 2 + 3 + 2z$$

Gauss's Law:

$$Q = \varepsilon_0 \iint_{\partial D} \bar{E}\cdot\bar{N}\,dS \qquad \bar{E} = \text{electric field.}$$

$E = \langle x, y, z\rangle$, $D =$ cube with corners $(\pm1, \pm1, \pm1)$

Every one of the 6 surface integrals was $4\varepsilon_0$, for a total of $24\varepsilon_0$.

$$Q = \varepsilon_0 \iiint_D \nabla \cdot \bar{E}\, dV \qquad \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle \cdot \langle x, y, z \rangle = 1+1+1 = 3$$

$$= \varepsilon_0 \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} 3\, dz\, dy\, dx = \varepsilon_0 3 \underbrace{\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} 1\, dz\, dy\, dx}_{\text{volume of the cube.}} = 3 \cdot 8\varepsilon_0 = 24\varepsilon_0$$