Assignment 25

Ex 16.3.2 Find an f so that $\nabla f = \langle x^3, -y^4 \rangle$, or explain why there is no such f. (answer)

P Q

$$P_y = 0 \qquad Q_x = 0$$

$$f_x = x^3 \qquad f = \frac{x^4}{4} + \underline{\;\frac{-y^5}{5}\;}$$

$$f_y = -y^4 \qquad f = -\frac{y^5}{5} + \underline{\;\frac{x^4}{4}\;}$$

$$f = \frac{x^4}{4} - \frac{y^5}{5}$$

Ex 16.3.3 Find an f so that $\nabla f = \langle xe^y, ye^x \rangle$, or explain why there is no such f. (answer)

P Q

$$\left.\begin{array}{l} P_y = xe^y \\ Q_x = ye^x \end{array}\right\} \text{not the same, so no such } f$$

$$f_x = xe^y \qquad f = \frac{x^2}{2}e^y + \underline{\qquad\qquad}$$

$$f_y = ye^x \qquad f = \frac{y^2}{2}e^x + \underline{\qquad\qquad}$$

no such f.

Ex 16.3.5 Find an f so that $\nabla f = \langle y\cos x, \sin x\rangle$, or explain why there is no such f. (answer)

$$\left.\begin{aligned} P_y &= \cos x \\ Q_x &= \cos x \end{aligned}\right\} \text{same}$$

$f_x = y\cos x \qquad f = y\sin x + \underline{\ 0\ }$

$f_y = \sin x \qquad f = y\sin x + \underline{\ 0\ }$

$f = y\sin x$

Ex 16.3.7 Find an f so that $\nabla f = \langle \overset{P}{yz}, \overset{Q}{xz}, \overset{R}{xy}\rangle$, or explain why there is no such f. (answer)

$P_y = z \qquad P_z = y \qquad Q_z = x$

$Q_x = z \qquad R_x = y \qquad R_y = x$

$f_x = yz \qquad f = xyz$

$f_y = xz \qquad f = xyz$

$f_z = xy \qquad f = xyz$

Ex 16.3.8 Evaluate $\int_C (10x^4 - 2xy^3)\,dx - 3x^2y^2\,dy$ where C is the part of the curve $x^5 - 5x^2y^2 - 7x^2 = 0$ from $(3,-2)$ to $(3,2)$. (answer)

$$\bar{r}(t) = ??? = \langle x(t), y(t) \rangle$$

$$\int_C 10x^4 - 2xy^3\,dx - 3x^2y^2\,dy = \int_C \langle 10x^4 - 2xy^3, -3x^2y^2 \rangle \cdot \langle dx, dy \rangle$$

$dx \to \frac{dx}{dt}dt \to x'dt$ $\qquad$ $dy \to \frac{dy}{dt}dt \to y'dt$

$$= \int_C \langle \underbrace{10x^4 - 2xy^3}_{P}, \underbrace{-3x^2y^2}_{Q} \rangle \cdot \langle x', y' \rangle\,dt$$

$$= \int_C \bar{F} \cdot \bar{r}'\,dt$$

$P_y = -6xy^2$

$Q_x = -6xy^2$

$f_x = 10x^4 - 2xy^3$

$f = \frac{10x^5}{5} - 2\frac{x^2}{2}y^3 = 2x^5 - x^2y^3 + \underline{\quad 0 \quad}$

$f_y = -3x^2y^2$

$f = -3x^2\frac{y^3}{3} = -x^2y^3 + \underline{\quad 2x^5 \quad}$

$f = 2x^5 - x^2y^3$

$$\int_C \bar{F} \cdot \bar{r}\,dt = 2x^5 - x^2y^3 \Big|_{(3,-2)}^{(3,2)} = 2\cdot 3^5 - 3^2 2^3 - (2\cdot 3^5 - 3^2(-2)^3)$$

$$= \cancel{2\cdot 3^5} - 3^2 2^3 - \cancel{2\cdot 3^5} + 3^2(-2)^3$$

$$= -3^2 2^3 - 3^2 2^3$$

$$= -9\cdot 8 - 9\cdot 8 = -72 - 72$$

$$= -144$$

Ex 16.3.9 Let $\mathbf{F} = \langle yz, xz, xy \rangle$. Find the work done by this force field on an object that moves from $(1,0,2)$ to $(1,2,3)$. (answer)

$f = xyz$

$$\int_C \langle yz, xz, xy \rangle \cdot \vec{r}\,' \, dt = xyz \Big|_{(1,0,2)}^{(1,2,3)}$$

$$= 6 - 0 = 6$$