Assignment 26

Ex 16.4.1 Compute $\int_{\partial D} \overset{P}{2y}\,dx + \overset{Q}{3x}\,dy$, where D is described by $0 \le x \le 1$, $0 \le y \le 1$. (answer)

$Q_x - P_y = 3 - 2 = 1$

$$\int_0^1\int_0^1 1\,dy\,dx = \int_0^1 y\Big|_0^1 dx$$

$$= \int_0^1 1\,dx = x\Big|_0^1 = 1$$

Ex 16.4.2 Compute $\int_{\partial D} \overset{P}{xy}\,dx + \overset{Q}{xy}\,dy$, where D is described by $0 \le x \le 1$, $0 \le y \le 1$. (answer)

$Q_x = y$, $P_y = x$

$$\int_0^1\int_0^1 y - x\,dy\,dx = \int \frac{y^2}{2} - xy\Big|_0^1 dx = \int_0^1 \frac{1}{2} - x\,dx$$

$$= \frac{1}{2}x - \frac{x^2}{2}\Big|_0^1 = \frac{1}{2} - \frac{1}{2} = 0$$

Ex 16.4.4 Compute $\int_{\partial D} \overset{P}{y\cos x}\,dx + \overset{Q}{y\sin x}\,dy$, where D is described by $0 \le x \le \pi/2$, $1 \le y \le 2$. (answer)

$Q_x = y\cos x \qquad P_y = \cos x$

$$\int_0^{\pi/2}\int_1^2 y\cos x - \cos x \;dy\,dx = \int_0^{\pi/2} \cos x\left(\frac{y^2}{2} - y\right)\Big|_1^2 dx$$

$$= \int_0^{\pi/2} \cos(x)\left[\cancel{2} - \cancel{2} - \left(\tfrac{1}{2} - 1\right)\right] dx$$

$$= \frac{1}{2}\int_0^{\pi/2} \cos x\,dx = \frac{1}{2}\sin x\Big|_0^{\pi/2} = \frac{1}{2}$$

Ex 16.4.7 Compute $\int_{\partial D} \overset{P}{(x/y)}\,dx + \overset{Q}{(2+3x)}\,dy$, where D is described by $1 \le x \le 2$, $1 \le y \le x^2$. (answer)

$Q_x = 3$

$P_y = x \cdot \frac{-1}{y^2} = \frac{-x}{y^2}$

$$\int_1^2\int_1^{x^2} 3 + \frac{x}{y^2}\;dy\,dx = \int_1^2 3y - \frac{x}{y}\Big|_1^{x^2} dx$$

$\int x y^{-2}\,dy = x\frac{y^{-1}}{-1} = -\frac{x}{y}$

$$= \int_1^2 3x^2 - \frac{x}{x^2} - 3 + \frac{x}{1}\;dx$$

$$= x^3 - \ln(x) - 3x + \frac{x^2}{2}\Big|_1^2$$

$$= 8 - \ln(2) - 6 + 2 - \left(1 - 3 + \tfrac{1}{2}\right)$$

$$= 8 - 6 + 2 - 1 + 3 - \tfrac{1}{2} - \ln(2)$$

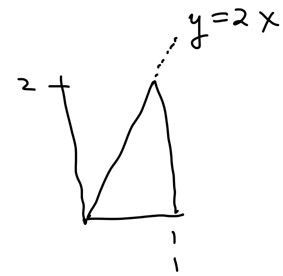
$$= \frac{11}{2} - \ln(2)$$

Ex 16.4.13 Evaluate $\oint_C (y - \sin(x))\,dx + \cos(x)\,dy$, where C is the boundary of the triangle with vertices $(0,0)$, $(1,0)$, and $(1,2)$ oriented counter-clockwise. (answer)

$P \qquad Q$

$Q_x = -\sin x$

$P_y = 1$



$$\int_0^1 \int_0^{2x} -\sin x - 1 \, dy\,dx$$

$$= -\int_0^1 (\sin x + 1)\, y \Big|_0^{2x} dx = -\int_0^1 2x(\sin x + 1)\, dx$$

$$= -2\int_0^1 x\sin x + x \, dx = -2\left[-x\cos x + \sin x + \frac{x^2}{2}\right]_0^1$$

$$= -2\left[-\cos(1) + \sin(1) + \frac{1}{2}\right]$$

$$= 2\cos(1) - 2\sin(1) - 1$$

$$\int x\sin x\,dx = -x\cos x + \int \cos x\,dx$$

$u = x \quad dv = \sin x\,dx \qquad = -x\cos x + \sin x$

$du = dx \quad v = -\cos x$