Assignment 27

Ex 16.4.6 Compute $\int_{\partial D} \overset{P}{x\sqrt{y}}\,dx + \overset{Q}{\sqrt{x+y}}\,dy$, where D is described by $1 \le x \le 2$, $2x \le y \le 4$. (answer)

$$\int_1^2 \int_{2x}^4 \frac{1}{2\sqrt{x+y}} - \frac{x}{2\sqrt{y}}\,dy\,dx = \int_1^2 \sqrt{x+y} - x\sqrt{y}\,\Big|_{2x}^{4} dx$$

$$= \int_1^2 \sqrt{x+4} - 2x - \sqrt{3x} + x\sqrt{2x}\,dx = \frac{2}{3}(x+4)^{3/2} - x^2 - \sqrt{3}\,\frac{2}{3}x^{3/2} + \sqrt{2}\,\frac{2}{5}x^{5/2}\Big|_1^2$$

$$= \frac{2}{3}6^{3/2} - 4 - \sqrt{3}\,\frac{2}{3}2^{3/2} + \frac{2\sqrt{2}}{5}2^{5/2} - \left(\frac{2}{3}5^{3/2} - 1 - \sqrt{3}\,\frac{2}{3} + \frac{2\sqrt{2}}{5}\right)$$

$$= \frac{8\sqrt{6}}{3} - \frac{10\sqrt{5}}{3} + \frac{2\sqrt{3}}{3} - \frac{2\sqrt{2}}{5} + \frac{1}{5}$$

Ex 16.4.9 Compute $\int_{\partial D} \overset{P}{x\ln y}\,dx$, where D is described by $1 \le x \le 2$, $e^x \le y \le e^{x^2}$. (answer)

$$\int_{\partial D} \overset{P}{x\ln y}\,dx + \overset{Q}{0}\cdot dy$$

$$\int_1^2 \int_{e^x}^{e^{x^2}} -\frac{x}{y}\,dy\,dx = \int_1^2 -x\ln y\,\Big|_{e^x}^{e^{x^2}} dx = \int_1^2 -x\ln(e^{x^2}) + x\ln(e^x)\,dx$$

$$= \int_1^2 -x\cdot x^2 + x\cdot x\,dx = -\frac{x^4}{4} + \frac{x^3}{3}\Big|_1^2 = -\frac{16}{4} + \frac{8}{3} + \frac{1}{4} - \frac{1}{3}$$

$$= -\frac{15}{4} + \frac{7}{3} = \frac{-17}{12}$$

Ex 16.4.10 Compute $\int_{\partial D} \overset{Q}{\sqrt{1+x^2}}\,dy$, where D is described by $-1 \le x \le 1$, $x^2 \le y \le 1$. (answer)

$$(1+x^2)^{1/2}$$

$$\int_{-1}^1 \int_{x^2}^1 \frac{2x}{2\sqrt{1+x^2}}\,dy\,dx = \int_{-1}^1 \int_{x^2}^1 \frac{x}{\sqrt{1+x^2}}\,dy\,dx = \int_{-1}^1 \frac{x}{\sqrt{1+x^2}}\,y\,\Big|_{x^2}^1 dx$$

$$= \int_{-1}^1 \frac{x}{\sqrt{1+x^2}}(1-x^2)\,dx = \int_{-1}^1 \frac{x}{\sqrt{1+x^2}} - \frac{x^3}{\sqrt{1+x^2}}\,dx = \int_{-1}^1 \frac{1-x^2}{\sqrt{1+x^2}}\,x\,dx$$

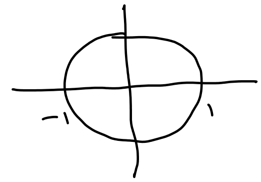
$$x^2 = u-1 \longleftarrow \quad u = 1+x^2, \quad du = 2x\,dx, \quad \frac{du}{2} = x\,dx$$

$$= \frac{1}{2}\int_2^2 \frac{1-(u-1)}{\sqrt{u}}\,du = \frac{1}{2}\int_2^2 \frac{2-u}{\sqrt{u}}\,du = 0$$

$$u = 1+1^2 = 2$$

$$u = 1+(-1)^2 = 2$$

Ex 16.4.11 Compute $\int_{\partial D} \overset{P}{x^2y}\,dx \underbrace{-\overset{Q}{xy^2}}\,dy$, where D is described by $x^2+y^2 \le 1$. (answer)



$$\iint_D -y^2 - x^2 \, dy\,dx = -\iint x^2+y^2 \, dy\,dx$$

$$= -\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} x^2+y^2 \, dy\,dx$$

$$= -\int_0^{2\pi}\int_0^1 r^2 \, r\,dr\,d\theta = -\int_0^{2\pi} \frac{r^4}{4}\Big|_0^1 d\theta = -\int_0^{2\pi} \frac{1}{4}\, d\theta$$

$$= -\frac{1}{4}\theta\Big|_0^{2\pi} = -\frac{1}{4}\cdot 2\pi = -\frac{\pi}{2}$$