Assignment 28

Ex 16.5.1 Let $\mathbf{F} = \langle \overset{P}{xy}, \overset{Q}{-xy} \rangle$ and let D be given by $0 \le x \le 1$, $0 \le y \le 1$. Compute $\int_{\partial D} \mathbf{F} \cdot d\mathbf{r}$ and $\int_{\partial D} \mathbf{F} \cdot \mathbf{N}\, ds$.

$$\int_{\partial D} \bar{F} \cdot d\bar{r} = \iint_D Q_x - P_y \, dy\,dx = \int_0^1\int_0^1 -y - x \, dy\,dx = -\int_0^1 \frac{y^2}{2} + xy \Big|_0^1 dx$$

$$= -\int_0^1 \frac{1}{2} + x \, dx = -\left(\frac{x}{2} + \frac{x^2}{2}\right)\Big|_0^1 = -1$$

$$\int_{\partial D} \bar{F} \cdot \bar{N}\, ds = \iint_D P_x + Q_y \, dy\,dx = \int_0^1\int_0^1 y - x \, dy\,dx = \int_0^1 \frac{y^2}{2} - xy \Big|_0^1 dx$$

$$= \int_0^1 \frac{1}{2} - x \, dx = \frac{x}{2} - \frac{x^2}{2}\Big|_0^1 = \frac{1}{2} - \frac{1}{2} = 0$$

Ex 16.5.2 Let $\mathbf{F} = \langle ax^2, by^2 \rangle$ and let D be given by $0 \le x \le 1$, $0 \le y \le 1$. Compute $\int_{\partial D} \mathbf{F} \cdot d\mathbf{r}$ and $\int_{\partial D} \mathbf{F} \cdot \mathbf{N}\, ds$.

$$\int_{\partial D} \bar{F} \cdot d\bar{r} = \int_0^1\int_0^1 0 - 0 \, dy\,dx = 0$$

$$\int_{\partial D} \bar{F} \cdot \bar{N}\, ds = \int_0^1\int_0^1 2ax + 2by \, dy\,dx = \int_0^1 2axy + 2b\frac{y^2}{2}\Big|_0^1 dx$$

$$= \int_0^1 2ax + b \, dx = 2a\frac{x^2}{2} + bx \Big|_0^1 = a + b$$

Ex 16.5.4 Let $\mathbf{F} = \langle \sin x \cos y, \cos x \sin y \rangle$ and let D be given by $0 \le x \le \pi/2$, $0 \le y \le x$. Compute $\int_{\partial D} \mathbf{F} \cdot d\mathbf{r}$ and $\int_{\partial D} \mathbf{F} \cdot \mathbf{N}\, ds$.

$$\int_{\partial D} \bar{F} \cdot d\bar{r} = \int_0^{\pi/2}\int_0^x -\sin x \sin y + \sin x \sin y \, dy\,dx = 0$$

$$\int_{\partial D} \bar{F} \cdot \bar{N}\, ds = \int_0^{\pi/2}\int_0^x \cos x \cos y + \cos x \cos y \, dy\,dx = \int_0^{\pi/2}\int_0^x 2\cos x \cos y \, dy\,dx$$

$$= 2\int_0^{\pi/2} \cos x \sin y \Big|_0^x dx = 2\int_0^{\pi/2} \cos x \sin x \, dx$$

$$= \sin^2 x \Big|_0^{\pi/2} = 1$$

Ex 16.5.5 Let $\mathbf{F} = \langle y, -x \rangle$ and let D be given by $x^2 + y^2 \leq 1$.

Compute $\int_{\partial D} \mathbf{F} \cdot d\mathbf{r}$ and $\int_{\partial D} \mathbf{F} \cdot \mathbf{N}\, ds$.

$$\int_{\partial D} \bar{F} \cdot d\bar{r} = \iint_D -1 - 1 \, dy\,dx = -2 \iint_D 1 \, dy\,dx = -2 \int_0^{2\pi} \int_0^1 r \, dr \, d\theta$$

$$= -2\pi$$

$$\int_{\partial D} \bar{F} \cdot \bar{N} \, ds = \iint_D 0 + 0 \, dy\,dx = 0$$