$f(x) = x^3 \qquad (f \circ g)(x) = f(g(x)) = (x^{1/3})^3 = x$

$g(x) = x^{1/3} \qquad (g \circ f)(x) = g(f(x)) = (x^3)^{1/3} = x$

Suppose $f: A \to B$, $g: B \to A$, $R = f(A)$

g is a pseudoinverse of f if for every $b \in R$

$f(g(b)) = b$. $g(b)$ is some preimage of b under f.

$f(x) = \sin(x) \qquad \sin(\arcsin x) = x$

$g(x) = \arcsin(x) \qquad \arcsin(\sin(x)) \neq x$

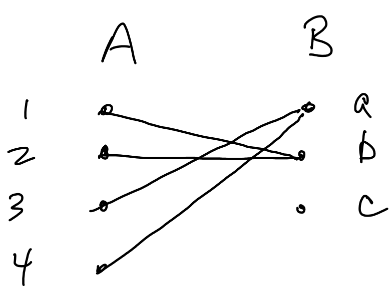
$\sin(\pi) = 0$

$\arcsin(0) = 0$

$\arcsin(\sin \pi) \neq \pi$.

Not always true that $g(f(a)) = a$.

If $b \in B$ but B is not in R, $g(b)$ can be anything in A.



$f(A) = \{a, b\}$

$g(a) = 3$ or 4

$g(b) = 1$ or 2

$g(c) = 1, 2, 3$ or 4

THM If f is 1-1, any pseudo-inverse of f, g, is onto.

If f is onto, any pseudoinverse of f, g, is 1-1.

Pf/ Suppose f is 1-1. Then every b in $f(A)$ has one preimage. $g(f(a)) = a$. So $g \circ f$ is onto. (4.4.1) Since $g \circ f$ is onto, so is g.

If f is onto, $f(A) = B$. For any $b \in B$, $f(g(b)) = b$. $(f \circ g) = i_B$. i_B is 1-1, so $f \circ g$ is 1-1. So g is 1-1.

If $g(f(a)) = a$ for all $a \in A$ then g is a left inverse of f.

If $f(g(b)) = b$ for all $b \in B$ then g is a right inverse.