

SAMPLE EXAM 4

1. Compute $\int_C xy \, ds$, where C is given by $\langle \sin \theta, \cos \theta \rangle$, $0 \leq \theta \leq \pi/2$.
2. Explain how you can tell that $\mathbf{F} = \langle 3x^2 \cos y, -x^3 \sin y \rangle$ is conservative. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is $\langle \cos t, t^2 \rangle$, $0 \leq t \leq 1$.
3. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} = \langle x^2 y^2, 3x + xy \rangle$ and C is the square $(0, 0) \rightarrow (1, 0) \rightarrow (1, 1) \rightarrow (0, 1) \rightarrow (0, 0)$.
4. Compute $\nabla \times \mathbf{F}$, $\mathbf{F} = \langle x \cos z, y \cos z, \sin z \rangle$. Is \mathbf{F} conservative?
5. Compute $\nabla \cdot \mathbf{F}$, $\mathbf{F} = \langle x \cos z, y \cos z, \sin z \rangle$.
6. Set up a double integral for the surface area of $\mathbf{r} = \langle u^2, u^2 - v, v^3 \rangle$, $0 \leq u \leq 1$, $0 \leq v \leq 1$.
7. Compute $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F} = \langle y, z, x \rangle$ and S is the surface $z = x^2 + y^2$ above the interior of the square with corners $(0, 0)$, $(1, 0)$, $(1, 1)$, $(0, 1)$.
8. Compute $\iint_S \nabla \times \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F} = \langle z, y, x \rangle$ and S is the surface $z = x^2 + y^2$ above the disk $x^2 + y^2 \leq 1$.
9. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} = \langle z^2, y, x \rangle$ and C is the triangle $(1, 0, 0) \rightarrow (0, 1, 0) \rightarrow (0, 0, 1) \rightarrow (1, 0, 0)$.
10. Compute $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F} = \langle x^2 z, z^2 y, y^2 x \rangle$ and S is the surface of the cube with corners $(0, 0, 0)$, $(1, 0, 0)$, $(1, 0, 1)$, $(0, 0, 1)$, $(0, 1, 0)$, $(1, 1, 0)$, $(1, 1, 1)$, $(0, 1, 1)$, oriented outward.