

Whitman College  
Econ 328  
Final Exam  
December 16, 2010

Write all answers in your blue book. Show **all of your work**, including all of the calculations. The exam ends at 5pm.

1. (a) (6pts) A formal definition of a game consists of six elements. List these six elements.

(b) (4pts) Game theory assumes that players act rationally. What does acting rationally mean in game theory?

2. Consider the following situation. There are three players, numbered 1, 2, and 3. Player 1 has two cards, labeled King and Ace. At the beginning of the game, Player 1 deals one of the cards to Player 2 and the other card to Player 3. That is, Player 1 either gives the Ace to Player 3 and the King to Player 2 (call this action A) or the King to Player 3 and the Ace to Player 2 (call this action K). Player 3 does not get to see the card dealt to her. Player 2 does get to see the card dealt to him. After looking at his card, Player 2 decides whether or not to switch his card with Player 3's card. Player 3 observes whether Player 2 makes the switch, but still does not get to see her card. Finally, Player 3 responds to the question "Is your card the Ace?" by saying either yes or no. If Player 3 correctly states whether her card is the Ace, then she gets a payoff of 1 and the other players get 0; otherwise, Players 1 and 2 both get a payoff of 1, and Player 3 gets 0.

(a) (10pts) Draw the extensive form for this situation. Be sure to label who plays at each decision node and what action each branch represents. Clearly indicate information sets. Make your diagram large and draw it carefully.

(b) (2pts) Is this a game of perfect or imperfect information? Explain how you can tell.

(c) (3pts) List the strategies in  $S_1$ , the strategy set for Player 1.

(d) (3pts) List the strategies in  $S_2$ , the strategy set for Player 2.

(e) (3pts) List the strategies in  $S_3$ , the strategy set for Player 3.

(f) (2pts) Write one strategy profile for this game.

(g) (1pt) Write the payoff profile associated with the strategy profile in part (f).

(h) (1pt) How many proper subgames does this game have?

3. (a) (5pts) Define a finite game

(b) (5pts) Define a pure strategy Nash equilibrium.

(c) (5pts) True or false? Every finite game has a pure strategy Nash equilibrium. If true, explain why it's true. If false, provide a counter example.

(d) (5pts) Define a mixed strategy Nash equilibrium.

(e) (5pts) True or false? Every finite game has a mixed strategy Nash equilibrium. If true, explain why it's true. If false, provide a counter example.

4. (20pts) Consider a location game like that discussed in class, between two soda vendors who each choose where to locate along a beach composed of nine linear segments filled with equal numbers of potential customers. Customers buy from the vendor closest to them. If both vendors locate in the same segment, they split the customers evenly. Consider the version of this game in which the vendors' wages do not depend on how many sodas they sell. Because the process of selling sodas requires effort, they'd rather sell as few cans as possible. Suppose that this game is played sequentially, so that Vendor 1 locates first, and then Vendor 2 locates, knowing where Vendor 1 has located. Find the subgame perfect Nash equilibria.

5. Consider a situation of strategic interaction between two friends, Lance and Bradley, who are considering going on vacation together. From vacationing together, Lance would get utility of 120 and Bradley would get utility of 40. If they didn't go on vacation together, each would get utility of 10. Assume that utility can be transferred 1:1 from one friend to another via side payments.

(a) (5pts) What would be the joint surplus from an agreement to go on vacation together?

Suppose that Lance proposes that they go on vacation, and makes an offer to Bradley of how to split the surplus between them. Bradley can accept the offer, or he can make a counteroffer of how to split the surplus between them. Assume that the alternating series of counteroffers could continue indefinitely. Lance has a discount factor of 0.90. Bradley has a discount factor of 0.85.

(b) (15pts) In equilibrium, what share of the surplus does Bradley get?

Consider the standard bargaining solution for the situation described above.

(c) (10pts) What utility does each friend get?

(d) (5pts) Who makes a side payment and what size is the side payment?

(e) (15pts) What utility would Lance have gotten if he had moved second rather than first?

5. Consider a one-shot tariff-setting game between two countries. Let  $x_1$  be the tariff level of Country 1 and  $x_2$  be the tariff level of country 2.

Country 1's payoff is  $4000 + 50x_1 + x_1x_2 - (x_1)^2 - 100x_2$ .

Country 2's payoff is  $4000 + 80x_2 + x_2x_1 - (x_2)^2 - 100x_1$ .

Assume that tariffs cannot be negative, and that countries set tariff levels simultaneously and independently. Note that the situation is not symmetric.

- (a) (15pts) Find the best response functions for each country.
- (b) (10pts) Compute the Nash equilibrium (NE).
- (c) (5pts) Show that if each country chooses a zero tariff level, both would be better off than they are in the NE.
- (d) (5pts) Is the NE a bad Nash? Explain your reasoning.

Now consider an infinitely repeated tariff-setting game, where the stage game is the one-shot tariff game described above. Suppose that Country 1 has a discount factor of 0.8, and Country 2 has a discount factor of 0.6.

- (e) (25pts) Does the grim trigger strategy support choosing zero tariff levels in each stage as a Nash equilibrium of the supergame? Explain your reasoning.

6. (a) (5pts) Describe the winner's curse in a common value auction.

(b) (5pts) Suppose six firms are bidding in a common value auction, for the rights to an exclusive contract. The value of this contract is uncertain, but each of the firms in the auction has an estimate of the value. These estimates are randomly drawn from a uniform distribution over a range centered on the true value of the contract. The range is  $\pm 50$ .

What is the optimal bid for a risk neutral firm that has an estimate of 179? Explain how the firm determines its optimal bid.