

Write all answers on the exam. **Show all of your work.** The exam ends at noon.

1. Consider the following location game between two players. Player 1 and Player 2 are the two candidates running in an election. The player with the most votes will win the election. Each player prefers winning to tying, and tying to losing. Neither player cares how many votes she wins or loses by, just whether she wins, ties or loses. Voters' political positions are unevenly located along a 5-segment political spectrum in the following way.

Segment 1	Segment 2	Segment 3	Segment 4	Segment 5
Number of voters: 40	Number of voters: 10	Number of voters: 10	Number of voters: 20	Number of voters: 20

Candidates simultaneously and independently choose which segment to locate in. They can locate in the same segment. So, the strategy sets for the players are  $S_1 = \{1, 2, 3, 4, 5\}$  and  $S_2 = \{1, 2, 3, 4, 5\}$ . Voters will vote for the candidate with the political position closest to the voter's political position. If both candidates are equally close, the voters will split evenly between them.

(a) (10pts) Fill out Player 1's best response sets below.

$$BR_1(1) = \{$$

$$BR_1(2) = \{$$

$$BR_1(3) = \{$$

$$BR_1(4) = \{$$

$$BR_1(5) = \{$$

(b) (5pts) Find all of the pure strategy Nash equilibria for this game.

2. Consider again the location game described Question 1, with the modification that the voters in Segment 1 will vote only for a candidate in Segment 1. If both candidates are in Segment 1, the Segment 1 voters will split evenly between them. In all other respects, the game is the same as that described in Question 1.

Segment 1	Segment 2	Segment 3	Segment 4	Segment 5
Number of voters: 40	Number of voters: 10	Number of voters: 10	Number of voters: 20	Number of voters: 20

(a) (10pts) Fill out Player 1's best response sets below.

$$BR_1(1) = \{$$

$$BR_1(2) = \{$$

$$BR_1(3) = \{$$

$$BR_1(4) = \{$$

$$BR_1(5) = \{$$

(b) (5pts) Find all of the pure strategy Nash equilibria for this game.

2. Consider the normal form below for a 3-player simultaneous-move one-shot game, in which  $S_1 = \{U, M, D\}$ ,  $S_2 = \{L, C, R\}$  and  $S_3 = \{A, B\}$ . Note that Player 3 is choosing either Matrix A or Matrix B. Recall that in the payoff profiles, Player 1's payoff is listed first, then Player 2's and then Player 3's.

		Player 3					Player 3		
		A					B		
		Player 2					Player 2		
		L	C	R			L	C	R
Player 1	U	4, 0, 5	3, 2, 0	6, 4, 1	Player 1	U	4, 8, 7	3, 3, 1	5, 2, 2
	M	8, 1, 6	5, 1, 0	3, 4, 2		M	4, 5, 9	4, 1, 1	6, 7, 3
	D	1, 3, 2	4, 2, 3	2, 1, 2		D	3, 2, 1	2, 3, 2	5, 1, 4

(a) (5pts) List the Pareto efficient strategy profiles for this game.

(b) (10pts) List all dominated strategies for this game. For each dominated strategy, list what strategy dominates it.

(c) (10pts) Find the set of rationalizable strategies for this game. Show and explain each step of your work.

(d) (15pts) Find all the pure strategy Nash equilibria of this game. Show your work. The game also appears on the next page, so you can show your work there.

Player 3

A

B

		Player 2		
		L	C	R
Player 1	U	4, 0, 5	3, 2, 0	6, 4, 1
	M	8, 1, 6	5, 1, 0	3, 4, 2
	D	1, 3, 2	4, 2, 3	2, 1, 2

		Player 2		
		L	C	R
Player 1	U	4, 8, 7	3, 3, 1	5, 2, 2
	M	4, 5, 9	4, 1, 1	6, 7, 3
	D	3, 2, 1	2, 3, 2	5, 1, 4

3. Two players will simultaneously and independently decide what nonnegative contribution to make to a public good. If Player 1 contributes  $x_1$  and Player 2 contributes  $x_2$ , then the value of the public good is  $2(x_1 + x_2 + x_1 x_2)$ , which they each receive. Player 1 must pay a contributing cost of  $(x_1)^2$ . So, Player 1's payoff in the game is  $u_1 = 2(x_1 + x_2 + x_1 x_2) - (x_1)^2$ . Player 2 pays the cost  $2(x_2)^2$  so that Player 2's payoff is  $u_2 = 2(x_1 + x_2 + x_1 x_2) - 2(x_2)^2$ .

(a) (15pts) Find the best response functions of each player.

(b) (15pts) Find the Nash equilibrium of this game. You can use the next page to show your work.