The SVD and QSVD in Signals Processing

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Goal: Present nice applications of the SVD and QSVD that are suitable for classroom use and give intuition about the kinds of information these factorizations can give.

All of these examples are easy to code in Matlab.

Outline:

- The SVD
  - Application 1: Low Rank Image Compression
  - Application 2: Understanding PCA
  - Application 3 (PCA): Face Recognition (Eigenfaces)
  - Application 4 (PCA): Temporal v. Spatial Coding
- The QSVD and Generalized Eigenvectors
  - Application 5: Max Noise Fraction
  - Application 6: Blind Signal Separation

The Singular Value Decomposition:

$$X = U\mathbf{\Sigma}V^T$$

$X \in \mathbf{R}^{m\times n}$
$U \in \mathbf{R}^{m\times m}$ (Basis for Col(X), Null(X^T))
$\mathbf{\Sigma} \in \mathbf{R}^{m\times n}$ (Diag)
$V \in R^{n\times n}$ (Basis for Row(X), Null(X))

The Thin SVD: (Rank of X is k)

$$X = \tilde{U}\tilde{\mathbf{\Sigma}}\tilde{V}^T$$

$X \in \mathbf{R}^{m\times n}$
$\tilde{U} \in \mathbf{R}^{m\times k}$ (Basis for Col(X))
$\tilde{\mathbf{\Sigma}} \in \mathbf{R}^{k\times k}$ (Nonzero, Diag)
$\tilde{V} \in R^{n\times k}$ (Basis for Row(X))

Application 1
Low Rank Approximations of Images

Example: A 320×200 pixel image- think of it as a matrix.

```
load clown;  %Loads X, map
[U,S,V]=svd(X);

%Normalized Singular Vectors
ss=diag(S)./sum(diag(S));
semilogy(ss,'k-*');

%Rank one approximations using pth vector:
A=S(p,p)*U(:,p)*V(:,p)';

%Rank k approximations:
A=U(:,1:k)*S(1:k,1:k)*V(1:k)';
```