Application 2
Understanding PCA

The Principle Components are the eigenvectors of the covariance of the data...

Let $X = U\mathbf{\Sigma}V^T$, $X \in \mathbf{R}^{n\times t}$ (column changing in time)

- $XX^T = U\mathbf{\Sigma}^2U^T$
  - Average over time (Cols of U in $\mathbf{R}^n$)
  - Eigenvecs of XX^T =Left SVs

- $X^TX = V\mathbf{\Sigma}^2V^T$
  - Average over space (Cols of V in $\mathbf{R}^t$)
  - Eigenvecs of X^TX =Right SVs

Application 2
Understanding PCA, continued

We also have:

$$XV = U\Sigma, \qquad X^T U = V\Sigma$$

When computing the PCA, we have a choice- either U or V.

Features of the PCA (or Karhunen-Loéve):

- Orthogonal
- Optimal (least squares) low dim rep.
- The eigs are stationary vals to

$$\phi^T X^T X\phi, \text{ s.t. } \|\phi\| = 1$$

- Data is uncorrelated in this basis.

Application 3
Face Coding (Eigenfaces)*

- Each face: 64×64 pixels, in $\mathbf{R}^{4096}$ (commercial grade: 128×128, $\mathbf{R}^{16,384}$)

- 10 images total (commercial grade: State DMV)

- PCA Eigs in $\mathbf{R}^{4096}$: Each face is coded by its eigenvector basis coefficients

- FaceIt technology: Based on this idea, but uses sub-block coding

*Data from M. Kirby, who received it from someone at MIT