Homework Compilation
Chapter 11

This includes the homework previously assigned since Oct 16th, plus some extra material.

1. (Lyapunov Exponents) If p is a fixed point for f, what is the Lyapunov exponent for any point whose orbit is attracted to p?

2. (Sect 11, Exercise 1) Can a continuous function on $\mathbb{R}$ have a periodic point of period 48 and not one of period 56? Why?

3. (Sect 11, Exercise 3) Give an example of a function $F : [0,1] \rightarrow [0,1]$ that has a periodic point of period 3 and NO other periods (Hint: Can this happen?).

4. (Sect 11, Exercise 4) Let f, g be piecewise linear functions where each has a cycle of period 4 given by $\{0,1,2,3\}$. In particular, define f as:

   $$f(0) = 1 \quad f(1) = 2 \quad f(2) = 3 \quad f(3) = 0$$

   and g:

   $$g(0) = 3 \quad g(1) = 2 \quad g(2) = 0 \quad g(3) = 1$$

   One of these functions has cycles of all other periods, and one has only periods $1, 2, 4$. Which is which? (They are easy to plot, but the graphs are given in Fig 11.16)

5. (Added from the board) If f is piecewise linear so that $f(1) = 2$, $f(2) = 3$ and $f(3) = 1$, find the interval (explicitly) that contains a period 4 point (The idea is to run through the proof of the Period 3 theorem to find intervals I_0, I_1, A_1, A_2, A_3, and A_4).

6. (Sect 11, Exercise 7) If f is piecewise linear so that

   $$f(0) = 4 \quad f(1) = 5 \quad f(2) = 3 \quad f(3) = 0 \quad f(4) = 1 \quad f(5) = 2$$

   (See Fig 11.17(a)), prove that there is a cycle of period 6 but no cycles of any odd period.

7. What was so special about *Period 3*? Why not period 4? Period 7?

8. Let F map points on the circle to the circle by rotating them $\pi/3$ radians.

   (a) Are there any period 3 points?

   (b) Are there any other (prime) periodic points?

   (c) Does this contradict the Period Three theorem?

9. Show that, if f is differentiable and decreasing for all x, then f^3 is decreasing.

10. Show that, if f is strictly decreasing and there is one fixed point of f, then there can be no other fixed points.

11. Define Σ_a as the subset of Σ_3 (the space of all symbol strings containing $0, 1$ or 2) where there are no double integers in the symbol string- That is, Σ_a cannot have 00, 11 or 22.

    (a) Show that Σ_a is closed in Σ (Prove that the complement is open- Look at how we did it in class)

    (b) Is Σ_a dense in Σ_3?

    (c) Is there a point in Σ_a whose orbit under σ is dense (in Σ_a)?

12. If f is continuous, and A is a closed interval, show that if $f(A) \supset A$, then A contains a fixed point of f.

13. Consider the following subsets of Σ_2 (the space of all symbol strings containing either 0 or 1). Is either dense in Σ_2?

    (a) $T_1 = \{(s_0 s_1 s_2 s_3 0 s_5 s_6 \cdots)\}$

    (b) $T_3 = \{(s_0 s_1 s_2 \cdots) \mid \text{ The sequence ends in all } 0's\}$

14. Let Σ' be the set of all symbol strings of zeros and ones so that 0 is always followed by 1. Show that σ on Σ' is chaotic.

15. Show that $\sigma : \Sigma \to \Sigma$ is continuous (using our usual metric d on Σ)

16. Let $F : \Sigma \to \Sigma$ so that

$$F(s_0 s_1 s_2 s_3 \cdots) = (s_0 s_2 s_4 s_5 \cdots)$$

    Is F continuous on Σ?