

Example Write Up: Experiment 5.6

In this experiment, we consider the orbit of $x_0 = 0.2$ under the mapping $F(x) = x^2 + c$ (for different values of c). We see that generally speaking, the fixed points are:

$$x = x^2 + c \quad \Rightarrow \quad x^2 - x + c = 0 \quad \Rightarrow \quad x = \frac{1 \pm \sqrt{1 - 4c}}{2}$$

The general derivative is $F'(x) = 2x$ for all values of c .

The table below summarizes the behavior that we saw, and checks the answer against the analysis of this chapter. The graphical analysis tells us that the orbits are all moving towards the left fixed point (needed for the computer algorithm- These are stored as `fixedpts`).

- $c = 1/4$ so the fixed point is $x = \frac{1}{2}$.

It took 99,987 iterations to get within 10^{-5} of the fixed point, and the derivative at the fixed point is 1 (neutral).

- $c = 0$ so the left fixed point is $x = 0$.

It took three iterations to get within 10^{-5} , and the derivative at the fixed point is 0.

- $c = -0.24$ so the left fixed point is $x = -0.2$

It took 1 iteration to reach the fixed point, but $F'(x) = -0.2$ (and the size of the derivative here is 0.4). If we try a different orbit, we get something better. If $x = 0.21$, it takes 8 iterations to get within 10^{-5} to the fixed point.

- $c = -3/4$, so the left fixed point is $x = -1/2$

In this case, the size of the derivative is 1, and after 100,000 iterations, the sequence had not yet converged (distance was 0.002238).