Homework: Chapters 1 and 2

Exercises for the Reading

1. You can answer the following questions by finding the appropriate passage in Chapter 1:

   (a) According to our author, what was the one major flaw in the (early) development of dynamical systems?

   (b) What was Sir Isaac Newton's original motivation for developing Calculus?

   (c) What is the $n-$body problem (from the text)? Go to the internet to see if you can find cool applets for the 3-body problem.

   (d) There was a time at which the study of dynamics languished, but for two notable exceptions: (Fill in the rest)

2. **Internet Exercises:**

   (a) What is the Fields Medal? While you're looking it up, see if you can find something out about Grigori Perelman, who declined the medal in 2006.

   (b) Steve Smale won the Fields Medal for his work in dynamical systems theory (see page 7, about three paragraphs down). Find Smale's website and see what he's been up to- When did he win the medal? See if you can find his biographical paper about finding the horseshoe on the beaches of Rio.

   (c) Pierre Fatou and Gaston Julia were instrumental in understanding Julia sets. One of them lost his nose in World War I and had to wear a leather strap across his face for the rest of his life. Which one was it?

Sequences, Limits and Convergence

In class, we will be iterating a function that will create a sequence of real numbers. The remaining questions below are from Calculus II to remind us about sequences, limits and convergence.

1. Look up the definition of the *limit*:

2. Determine whether or not the following sequence converges or diverges. If it converges, find the limit:

   (a) $a_n = \cos(2/n)$

   (b) $a_n = \dfrac{(-1)^n n}{n^2+1}$

   (c) $\{1, 2, -1, 1, 2, -1, 1, 2, -1, \ldots\}$

3. Recall that if the limit of a sequence exists, $\lim_{n\to\infty} a_n = L$, then $\lim_{n\to\infty} a_{n+1} = L$ as well. Use this to find the limit of each sequence (you may assume each sequence converges):

   (a) $a_{n+1} = 1/(1+a_n)$

   (b) $a_{n+1} = \dfrac{1}{2}(a_n + 6)$

   (c) $a_{n+1} = \dfrac{1}{2}\left(a_n + \dfrac{c}{a_n}\right)$, where $c > 0$ (c is fixed- Your limit will depend on c).

An illustrative example: Find the limit, if we assume the sequence a_n converges: $a_{n+1} = \sqrt{1+3a_n}$

SOLUTION: Let L be the limit of the sequence, so that

$$\lim_{n\to\infty} a_n = L \qquad\qquad \lim_{n\to\infty} a_{n+1} = L$$

Then:

$$\lim_{n\to\infty} \sqrt{1+3a_n} = \sqrt{\lim_{n\to\infty}(1+3a_n)} = \sqrt{1+3\lim_{n\to\infty} a_n} = \sqrt{1+3L}$$

Thus, if the limit exists, then:

$$L = \sqrt{1+3L} \quad \Rightarrow \quad L^2 - 3L - 1 = 0 \quad \Rightarrow \quad L = \frac{3 \pm \sqrt{13}}{2}$$

Try each one, since we had to square the answer. If you do that, you will find that, if $L = (3-\sqrt{13})/2$, then

$$\sqrt{1+3L} = \frac{-3+\sqrt{13}}{2}$$

so we would discard this solution and keep the other. Try it and see, for example, with $a_1 = 2$. Then:

$$a_2 = 2.6548,\ a_3 = 2.989,\ \ldots a_{10} = 3.02\ldots \approx \frac{3+\sqrt{13}}{2}$$