Homework Questions: Chapters 3-5

Please write up the solutions to the following questions and turn them in on Monday, September 15.

1. (Similar to questions 12, 13 on p. 27) Let $D(x)$ be the doubling map,

$$D(x) = \begin{cases} 2x \text{ if } & 0 \leq x < \frac{1}{2} \\ 2x - 1 \text{ if } & \frac{1}{2} \leq x \leq 1 \end{cases}$$

   (a) Sketch the graph of $D(x)$ and graphically show the location of the fixed points, if any.

   (b) Algebraically determine the fixed points.

   (c) Sketch the graph of $D^2(x)$ and graphically show the period 2 points.

   (d) Find an algebraic formula for $D^2(x)$, and then determine the period 2 points (again, algebraically).

   (e) Determine (algebraically, not graphically) whether the fixed points in 1(b) and the period 2 points in 1(d) are attracting or repelling.

2. (These are some selected questions from Ch. 4, problem 7) Let $F(x) = ax + b$

   (a) For what value(s) of a, b does F have exactly 1 fixed point? No fixed points? An infinite number of fixed points?

   (b) If F has exactly one fixed point, for what value(s) of a, b will that fixed point be attracting?

3. A function $f : [a, b] \mapsto [a, b]$ is said to be a *contraction map* if

$$|f(x_1) - f(x_2)| < k|x_1 - x_2|$$

   where $k < 1$ and for any x_1, x_2 in the interval $[a, b]$. Show that, if f is a contraction map, then

$$|f^n(x_1) - f^n(x_2)| < k^n|x_1 - x_2|$$

   (Hint: The purpose of this exercise is to get you to look at pages 43 and 44 again)

4. Download the Matlab files for Experiment 5.6: Rates of Convergence. The solution for functions (a)-(d) are online. Change the code to get the solutions for functions (e)-(j).

   (Please start this one early enough so that you can ask me for help with the Matlab code if needed. You can email me and attach the code if you have specific questions about it)