Exercises for Chapter 10

1. Find the intervals that are the preimage of $[0, 1]$ if $f(x) = x^2 - 1$. Continuing, find the intervals that are the preimage of your previous answer.

2. Prove that the set of endpoints of removed intervals in the Cantor middle thirds set is a dense subset of the Cantor set.

3. Show that the dynamical system $f(x) = 2x$ on $[0, \infty)$ is sensitive to initial conditions. Is the dynamical system transitive?

4. Show that the dynamical system $f(x) = \frac{1}{2}x$ on $[0, \infty)$ is not sensitive to initial conditions. Is the dynamical system transitive?

5. Exercise 21, p. 132: Prove that the function

$$T(x) = \begin{cases} 2x & \text{if } x \leq 1/2 \\ 2 - 2x & \text{if } x > 1/2 \end{cases}$$

   is chaotic on $[0, 1]$. We want to prove this directly, as we did in class (not via a homeomorphism).

6. Anticipating some things in Chapter 12, we define an operator on a function F: The *Schwarzian derivative* of a function F is:

$$SF(x) = \frac{F'''(x)}{F'(x)} - \frac{3}{2}\left(\frac{F''(x)}{F'(x)}\right)^2$$

   (a) Compute S, if $F(x) = e^x$.

   (b) Compute S, if $F(x) = \sin(x)$