Exercises, 9.1-9.2

1. Let x, y be real numbers. Show that

$$d(x, y) = 2|x - y|$$

   is a metric on the reals.

2. Let x, y be real numbers. Show that

$$d(x, y) = |xy|$$

   does NOT define a metric.

3. Let $f(x) = 1 - 2x$. Prove that f is continuous at $x = 1$.

4. (Optional) Prove that $x^2 - 1$ is continuous at $x = 1$.

5. Compute $d(s, t)$ for the following, where d is the metric on Σ as defined in our text:

   (a) $s = (\overline{100})$ $\quad t = (\overline{001})$

   (b) $s = (\overline{1011})$ $\quad t = (01\overline{01})$

6. Find the set of points in Σ whose distance from $(0000000\ldots)$ is exactly $1/2$.

7. Prove the statement directly (without the Proximity Theorem): (i) Any point in M_0 must be at least 1 unit away from any point in M_1. (ii) Any point in M_{00} must be at least $1/2$ units from M_{01}.

8. Give an example of a sequence midway between $(000\cdots)$ and $(111\cdots)$. (Are there only 2?)