Algebra Practice Problems

1. For each expression, factor out the greatest common factor.

   Example: $18x^5 + 6x^4 + 24x^3$.

   SOLUTION: Factor each term first, then select common factors:

   $$2 \cdot 3 \cdot 3 \cdot xxxxx - 2 \cdot 3 \cdot xxxx + 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot xxx$$

   There is a common 2, a common 3, then three common x's, so we can factor out $6x^3$:

   $$18x^5 + 6x^4 + 24x^3 = 6x^3(3x^2 + x + 4)$$

   (a) $6px^2 - 8px^3 - 12px$

   (b) $60a^4b^3 - 15a^3b^4 + 45a^2b^5$

   (c) $2(a + b) + 4m(a + b)$

2. Practice multiplying polynomials: Expand each expression.

   Example: $(1 + x)(2 - x) = 2 - x + 2x - x^2 = 2 + x - x^2$

   (a) $(2x + 3)(x - 5)$

   (b) $(x + 1)^2$

   (c) $(2x + 1)(3x^2 - x + 4)$

   (d) $(x - 2)(x + 2)$

3. In the last two computations of the previous problem, did you notice a pattern? This is called a difference of squares- The pattern is:

   $$a^2 - b^2 = (a - b)(a + b)$$

   For each expression below, factor as a difference of squares:

   (a) $9x^2 - 25$

   (b) $81x^2 - 100$

   (c) $4 - x$ (hint: $x = (\sqrt{x})^2$)

4. Factor each polynomial as $(x - a)(x - b)$.

   Notice that $(x - a)(x - b) = x^2 - (a + b)x + ab$, so when you look at a polynomial with a leading x^2, we need a, b so that the sum is the middle term and the product is the last term.

   A couple of examples:

   - $x^2 - 5x + 6$: We need two numbers that sum to -5 and whose product is 6. I think they are -2 and -3, so: $(x - 2)(x - 3)$ (you can multiply this out to check).
   - $x^2 - x - 6$: Sum is -1 and product is -6: The two numbers are -3 and +2.

   $$x^2 - x - 6 = (x + 2)(x - 3)$$

   Here are some for you to try:

   (a) $x^2 + x - 2$

   (b) $x^2 + x - 12$

   (c) $x^2 - x - 6$

(d) $x^2 - 16$

5. Write each expression in lowest terms:

   Example: $\dfrac{8x^2+16x}{4x^2}$

   SOLUTION: Factor, then we can cancel factors common to both numerator and denominator.

$$\frac{8x^2+16x}{4x^2} = \frac{8x(x+2)}{4x^2} = \frac{2(x+2)}{x}$$

   (a) $\dfrac{20r+10}{30r+15}$

   (b) $\dfrac{3t+15}{(t+5)(t-3)}$

   (c) $\dfrac{3t+15}{(t+5)(t-3)}$

   (d) $\dfrac{x^2-x-6}{x^2+x-12}$

   (e) $\dfrac{x^2-4x+4}{x^2+x-6}$

6. Rationalize something of the form $\sqrt{a}-b$

   Idea: To rewrite the expression $\sqrt{a}-b$ (and get rid of the square root in the numerator), multiply the expression by $(\sqrt{a}+b)/(\sqrt{a}+b)$. Example:

$$\sqrt{x}-a = \sqrt{x}-a\cdot\frac{\sqrt{x}+a}{\sqrt{x}+a} = \frac{x-a^2}{\sqrt{x}+a}$$

   (a) $\sqrt{1+h}-1$

   (b) $\sqrt{x+h}-\sqrt{x}$

   (c) $\sqrt{x+3}-2$

7. Let f, g be given. Compute $f\circ g$.

   (a) $f(x) = 2x-1$ and $g(x) = 3x+1$

   (b) $f(x) = 1/x$ and $g(x) = x^2$

   (c) $f(x) = \sqrt{x+1}$ and $g(x) = \frac{8}{x}$.

8. In each of the following h is given. Find functions f, g so that $h(x) = f(g(x)) = (f\circ g)(x)$.

   (a) $h(x) = (6x-2)^3$

   (b) $h(x) = \frac{1}{x^2+2}$

   (c) $h(x) = \frac{(x-2)^2+1}{5-(x-2)^2}$

   (d) $(x+2)^3 - 3(x-2)^2$