M124 Fall 2023

Midterm

Name:_____

Show all your work! You are allowed to use the one-half sheet of notes (one-sided) that you prepared for this exam. You are also allowed to use a calculator, but **only for numerical use, no graphs.**

- 1. If f(1) = 3 and f'(1) = -2, then estimate f(1.3) using the tangent line of f.
- 2. If $F(x) = \sqrt{1 + \sin(x)}$, find functions f(x), g(x) and h(x) so that $F = f \circ g \circ h$.
- 3. Solve for x (exact or as a numerical approximation, your choice):
 - (a) $\log_9(x) = \frac{3}{2}$

(b) $4^{3x-1} = 5$

(c) $3^{2\log_3(x)} = 9$

4. Solve $\sin(x) = -0.4$ using the arcsine, for all x in $[0, 2\pi]$ (you may use numerical approximations).

5. Find the limit (algebraically), if it exists: $\lim_{x \to 2} \frac{x^2 - 2x}{x^2 - 4}$

6. Using the **definition** of the derivative, find f'(4) if $f(x) = \sqrt{x}$

- 7. Sketch a function p that satisfies all of the following:
 - (a) p(-1) = 1, and $\lim_{x \to -1} p(x) = 0$
 - (b) p(0) = 1 and p'(0) = -1 and p''(0) > 0.
 - (c) p is continuous at x = 1, but is not differentiable there.

- 8. Sketch a right triangle with angle θ and length of the side adjacent is x, and the hypotenuse length is 1. Determine the simplest expression you can for each of the following in terms of x:
 - (a) $\csc(\theta) =$
 - (b) $\tan(\arccos(x)) =$
- 9. Find **exact** values of each of the following:
 - (a) $\arcsin(\sin(5\pi/6))$
 - (b) $\cot(2\pi/3)$
 - (c) $\cos(\arcsin(1/2))$
- 10. Given $f(t) = -3\cos(5t) + 6$,
 - (a) What is the amplitude of f?
 - (b) What is the midline of f?
 - (c) What is the period of f?

11. The curve below shows a function s(t), in feet, that measures displacement of an object at time t, measured is seconds. The dotted line shown is a tangent line to the curve.



(a) Find the average rate of change of s over the interval t = 0 to t = 3. Include units.

- (b) Find the instantaneous velocity at t = 2. Include units.
- (c) Find the equation of the tangent line (the dotted line in the graph).
- 12. A cup of warm water at 100 degrees C is placed in a room that has constant temperature 5 degrees C. Five minutes later, the temperature of the water is 75 degrees C. Use the model for heating and cooling to find a function for the temperature of the water at time t.