Math 125: Practice Final

These practice problems are largely comprised of problems from past Math 125 Final Exams. They are meant to represent the types of problems that you're likely to encounter on the final. Note that this set of problems is not exhaustive. Any inclusions or omissions here should not induce you to study more or fewer topics.

1. Suppose that a hiker's elevation, in feet, is modeled by $f(x) = 30x^2 + 50x$ after x hours
   (a) What is the average rate of elevation gain of the hiker between 1 and 2 hours?
   (b) What is the hikers instantaneous rate of elevation gain at $t = 1.5$ hours?
2. A water balloon has height modeled by $f(t) = -5t^2 + 20t + 25$, where height is measured in feet and time in seconds.
   (a) Find the maximum height hit by the water balloon.
   (b) Give a function for the velocity of the water balloon.
   (c) When the balloon hits the ground, how fast is it going?
3. A cup of coffee at 180 degrees (F) is placed in a room with temperature 72 degrees (F). After 10 minutes, the temperature is 135 degrees (F). Use the model for heating and cooling to find out when the temperature is 100 degrees (F).
4. An observer standing 20 feet from a building notes that the angle between the ground and the top of the building is 72 degrees. How tall is the building?
5. State the definition of the derivative. Use it to calculate the derivative of $f(x) = x^2 - \frac{1}{x}$
6. (a) Find $f(x)$ if $f'(x) = 3x^2 + \frac{1}{x}$
   (b) Find $f(x)$ if $f'(x) = 3x^2 + \frac{1}{x}$
7. Calculate $f'(x)$ for the following functions:
   (a) $f(x) = (x^2 + x)$
   (b) $f(x) = e^{x^2+x}$
   (c) $f(x) = \cos(e^{x^2+x})$
8. Let $f(x) = \sin(x^2)$. Calculate the first two derivatives of $f(x)$
9. Find the equation of the tangent line to $f(x) = \sqrt{x}$ at $x = 9$. Use this to approximate $\sqrt{8.9}$.
10. Suppose that $y^3 - xy = -6$. Find $\frac{dy}{dx}$, and find the equation of the tangent line to the curve at the point $(7, 2)$.
11. Draw a curve $f(x)$ that is bounded for all x, increasing and concave up for $x < a$, and decreasing and concave up for $x > a$. Can such a curve be differentiable at $x = a$? Explain.

12. The volume of a cube in increasing at a rate of 10 cm^3/min.

    (a) How fast is the length of a side increasing when that length is 30 centimeters?

    (b) How fast is the surface area changing at this time?

13. Calculate

$$\lim_{x\to 0} \frac{e^{2x} - x^2 - 1}{x^3 - x}$$

14. Student learning can be modeled by the function $F(S, C) = C^2 S$, where S is the number of hours of sleep a student gets, and C is the number of ounces of caffeine a student takes in. Since students can't sleep and drink caffeine as much as they'd like, we're bound by the equation $S + C = 24$. Find the values of S and C that maximize student learning. Be sure to justify why these give a *maximum* value. [1]

15. Consider $f(x) = x^3 - 9x^2 - 48x + 54$. Find all intervals where the graph is increasing, decreasing, and classify all critical points. Find all intervals where the graph is concave up, concave down, and any inflection points. Find the y-intercept. Plot the critical and inflection points and intercepts and draw a rough sketch of $f(x)$.

[1] As you pursue more mathematics, the exponent on C increases; as you age, the exponent on S does!