Quiz 5 Review Sol'ns.

1. <u>EVT</u>: If f is cont. on $[a,b]$, then f attains a global max/min on $[a,b]$.

2. To apply l'Hospital's Rule,

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \text{"}\frac{0}{0}\text{"} \text{ or } \text{"}\frac{\pm\infty}{\pm\infty}\text{"}.$$

In that case,

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

provided the last limit exists or is $\pm\infty$.

3. The Closed interval method (for continuous functions on a closed interval).

a) Find the critical points for f.

b) Build a table with endpoints, crit. pts.

	x	$f(x)$
endpts & c.pts		

Find the largest & smallest.

4 (a) $\lim\limits_{x \to 1} \dfrac{x^2-1}{x^2-x} = "\dfrac{0}{0}"$

$\overset{L}{=} \lim\limits_{x \to 1} \dfrac{2x}{2x-1} = \dfrac{0}{-1} = 0$

b) $\lim\limits_{x \to 1} \dfrac{x^2+5}{3x+4} = \dfrac{6}{7}$

c) $\lim\limits_{x \to 0} \dfrac{\arcsin(x)}{x} = "\dfrac{0}{0}"$

$\lim\limits_{x \to 0} \dfrac{\frac{1}{\sqrt{1-x^2}}}{1} = 1$

d) $\lim\limits_{x \to 0} \dfrac{x\,2^x}{2^x-1} = "\dfrac{0}{0}"$

$\lim\limits_{x \to 0} \dfrac{2^x + x \cdot 2^x \ln(2)}{2^x \ln(2)} = \dfrac{1}{\ln(2)}$

e) $\lim\limits_{x \to \infty} \dfrac{\ln(\sqrt{x})}{x^2} = "\dfrac{\infty}{\infty}"$

$\lim\limits_{x \to \infty} \dfrac{\ln(\sqrt{x})}{x^2} = \lim\limits_{x \to \infty} \dfrac{\ln(x^{1/2})}{x^2} = \lim\limits_{x \to \infty} \dfrac{\frac{1}{2}\ln x}{x^2}$

$\overset{L}{=} \lim\limits_{x \to \infty} \dfrac{\frac{1}{2x}}{2x} = \lim\limits_{x \to \infty} \dfrac{1}{2x} \cdot \dfrac{1}{2x} = 0.$

f) $\lim\limits_{x \to 0} x^3 e^{-x^2} = 0.$

5. $f(x) = \dfrac{x^2}{x-2}$, $[3, 7]$

a) C.P.s:

$$f'(x) = \frac{2x(x-2) - x^2 \cdot 1}{(x-2)^2} = \frac{x^2-4x}{(x-2)^2} = 0$$

$$\Rightarrow x^2 - 4x = 0 \text{ or } x(x-4) = 0 \Rightarrow \cancel{x=0},\ \underline{\underline{x=4}}$$

b) Table:

x	$f(x)$
3	9
4	8 ← min
7	$49/5 = 9.8$ ← max

The global max is 9.8, it occurs when $x=7$. The global min is 8 (and it occurs when $x=4$).

6. $f(x) = x^3 - 3x$.

$f'(x) = 3x^2 - 3$

$f'(x) = 0 \Rightarrow 3x^2 - 3 = 0$

$x = \pm 1$

$f''(x) = 6x$

$f''(x) = 0 \Rightarrow 6x = 0$

$x = 0$

		$\frown$		$\smile$	
$f'(x)$	$+$	-1	$-$	1	$+$

$f''(x)$	$-$	0	$+$

a) At $x=-1$, we have a local max, and at $x=1$, we have a local min. (First deriv test)

c) For part (c), we have an inflection point at $x=0$.

b)

x	$f(x)$
0	0 ← global min
1	-2
2	2 ← global max

$\}$ on $[0, 2]$.

7.) Try it and put in your answer to have it corrected.

8) Let x, y be the numbers; then we want to minimize the product: $P = xy$ such that the ~~sum~~ difference is 100: $x - y = 100$.

So $y = x - 100$

and min. $P(x) = x(x-100)$, x any real number.

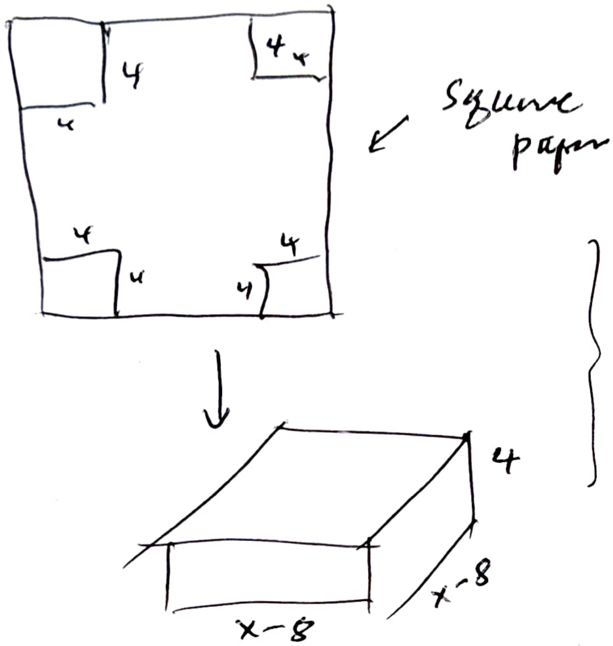
Solution 1: $P(x) = x^2 - 100x$ is a parabola opening upward - the min is at the vertex, halfway between 0 and 100 (so $x = 50$).

The two numbers are 50 and -50.

Solution 2: $P'(x) = 2x - 100 = 0 \Rightarrow x = 50$ is the only critical point.

Global min at $x = 50$ $(y = -50)$

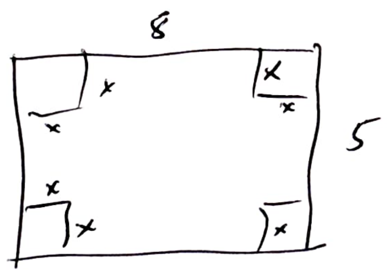
9.)



$$4(x-8)^2 = 784$$
$$(x-8)^2 = 196$$
$$x-8 = 14$$
$$\boxed{x = 22}$$

(Not a ~~c~~ calculus problem - practice with boxes!)

10.)



$V(x) = x(8-2x)(5-2x) \quad 0 \le x \le 2.5$

Expanding V,

$V(x) = 4x^3 - 26x^2 + 40x$

$V'(x) = 12x^2 - 52x + 40$

Using the quadratic formula, $x = 3\frac{1}{3}$ or $x = 1$.

Too big ... ok

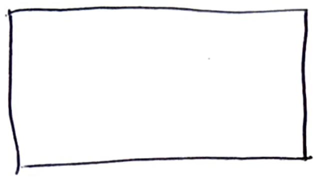
Build Table:

x	V(x)
0	0
1	18
2.5	0

max is found when x=1

The max is 18, it occurs when we cut squares of 1 inch from each corner.

11)



Let x, y be the length, width of the rectangle. Then $2x+2y$ is the perimeter, and xy is the area.

Max xy such that $2x+2y=100$

We substitute: $y = \frac{100-2x}{2} = 50-x$ in the expression.

max: $A(x) = x(50-x) = 50x - x^2$, $\quad 0 \le x \le 50$

So $A'(x) = 50-2x$ or $\boxed{x=25, \; y=25.}$

x	$A(x)$
0	0
25	$25^2 = \underline{625}$
50	0

12. $Y = \frac{kN}{1+N^2}$ $\quad (k>0)$ Find the N that maximizes Y.

Crit Points:

$$\frac{dY}{dN} = \frac{k(1+N^2) - kN(2N)}{(1+N^2)^2}$$

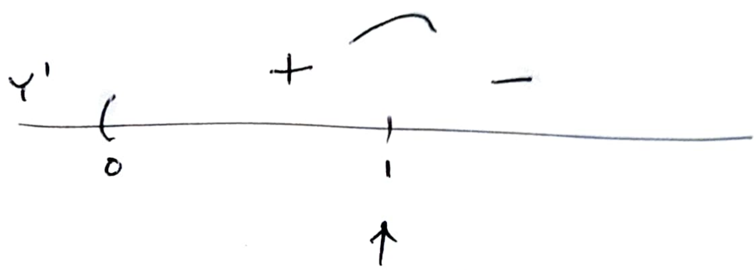
$$= \frac{k(1+N^2-2N^2)}{(1+N^2)^2} = \frac{k(1-N^2)}{(1+N^2)^2} = 0$$

$$\Rightarrow 1-N^2=0$$

$N = \pm 1$ (only use $+1$)

$\downarrow$

First deriv test



Global max at $x=1$.

And in that case, $y = \dfrac{k \cdot 1}{1+1} = \boxed{\dfrac{k}{2}}$.