M125
Fall 2024

Sample Final 1

Name:______________

I certify that the work on this exam is my own and that I have not discussed the specific contents of this exam with anyone prior to taking it.

Initials:

Instructions: No calculators are allowed, and no notes beyond what is provided here. Answers with no appropriate justification will receive no credit.

1. Find the domain of $f(x) = \dfrac{\sqrt{x^2 - 1}}{x - 2}$

2. Evaluate the following:

   (a) $\sin(5\pi/12)$ (b) $\tan^{-1}(1/\sqrt{3})$ (c) $\log_5(1/\sqrt[3]{25})$

3. True or False (and give a short reason):

   (a) If f is continuous at $x = a$, then f is differentiable at $x = a$.

   (b) If $3 \leq f(x) \leq 5$ for all x, then $6 \leq \int_1^3 f(x)\,dx \leq 10$.

   (c) All continuous functions have antiderivatives.

   (d) $\int_{-2}^{1} -x^{-2}\,dx = x^{-1}\big|_{-2}^{1} = \dfrac{3}{2}$

4. Find $f'(2)$ directly from the definition of the derivative, if $f(x) = \frac{1}{x}$ (you may not use l'Hospital's rule).

5. Compute the following limits, where you may not use l'Hospital's rule:

   (a) $\displaystyle\lim_{x \to 2} \frac{x^2 - x - 2}{x - 2}$ (b) $\displaystyle\lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$

6. Find the limit, if it exists (you may use any technique from class):

   (a) $\displaystyle\lim_{x \to -\infty} \sqrt{\frac{2x^2 - 1}{x + 8x^2}}$ (b) $\displaystyle\lim_{x \to 9} \frac{\sqrt{x}}{x - 9}$

7. Differentiate:

   (a) $\dfrac{2x}{\sqrt{x^2 + 1}}$ (b) $y = \sin^3(x^2 + 1)$ (c) $y = x^x$

8. Find the equation of the tangent line to $\sqrt{x + y} = \frac{1}{2}xy$ at the point $(2, 2)$.

9. Evaluate the Riemann sum by first writing it as an appropriate definite integral: $\displaystyle\lim_{n \to \infty} \sum_{i=1}^{n} \frac{3}{n} \sqrt{1 + \frac{3i}{n}}$.

10. Differentiate: $F(x) = \displaystyle\int_{\sqrt{x}}^{x^2} \frac{t}{1 + t}\,dt$

11. Write the definite integral as an appropriate Riemann sum (using right endpoints): $\displaystyle\int_0^3 1 + 3x^2\,dx$

12. Evaluate, or find the general indefinite integral.

(a) $\int \sqrt{x^3} + \frac{1}{x^2+1}\,dx$ (b) $\int_{-1}^{1} t(1-t)\,dt$ (c) $\int_0^1 5x - 5^x\,dx$

13. Evaluate:

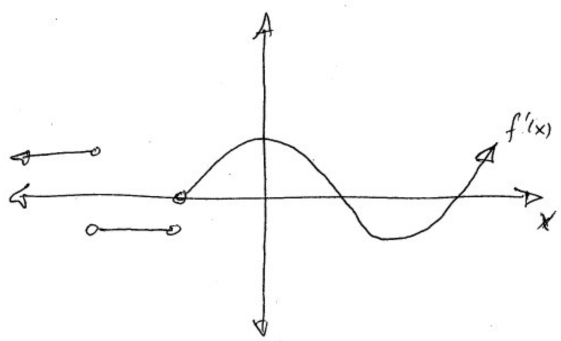
(a) $\int_0^1 \frac{d}{dx}\left(e^{\tan^{-1}(x)}\right)dx$ (b) $\frac{d}{dx}\int_0^1 e^{\tan^{-1}(x)}\,dx$ (c) $\frac{d}{dx}\int_0^x e^{\tan^{-1}(t)}\,dt$

14. Given the graph of the derivative, $f'(x)$, below, answer the following questions:

   (a) Find all intervals on which f is increasing.

   (b) Find all intervals on which f is concave up.

   (c) Sketch a possible graph of f if we require that $f(0) = -1$.



15. A rectangle is to be inscribed between the $x-$axis and the upper part of the graph of $y = 8 - x^2$ (symmetric about the $y-$axis). For example, one such rectangle might have vertices: $(1,0), (1,7), (-1,7), (-1,0)$ which would have an area of 14. Find the dimensions of the rectangle that will give the largest area.

16. Find all values of c and d so that f is continuous at all real numbers:

$$f(x) = \begin{cases} 2x^2 - 1 & \text{if } x < 0 \\ cx + d & \text{if } 0 \le x \le 1 \\ \sqrt{x+3} & \text{if } x > 1 \end{cases}$$

Be sure it is clear from your work that you understand the definition of continuity.

17. A perfectly round balloon is being inflated at a rate of 2000 cubic cm pepr second. How fast is the radius of the balloon increasing when the radius is 10 cm? (Volume of a sphere is $V = \frac{4}{3}\pi r^3$).

18. Suppose that over 45 months, the amount of radioactive element in a sample goes from 4 grams to $\frac{1}{2}$ gram. (i) Find a formula for the amount present at any time t. (ii) What is the half-life of the element? (Leave answers in exact form)