<div align="center">Sample Final 2</div>

Name:_____

I certify that the work on this exam is my own and that I have not discussed the specific contents
of this exam with anyone prior to taking it.

Initials:

Instructions: No calculators are allowed, and no notes beyond what is provided here. Answers with no
appropriate justification will receive no credit.

1. Short Answer:

    (a) True or False? $\dfrac{x^2 - 1}{x - 1} = x + 1$

    (b) If $f'(2)$ exists, then $\lim\limits_{x \to 2} f(x) = f(2)$

    (c) If $f(x) = (2 - 3x)^{-1/2}$, find $f(0)$, $f'(0)$ and $f''(0)$.

    (d) Show that the $x^4 + 4x + c = 0$ has at most one solution in the interval $[-1, 1]$.

2. Differentiate:

    (a) $y = xe^{3x}$.

    (b) $y = 4^{1/x} + \sin^{-1}(3x + 1)$

    (c) $y = \dfrac{x^2 - 1}{\sqrt{x}}$

3. Find the equation to the tangent line for $\sqrt{x} + \sqrt{y} + xy = 3$ at the point $(1, 1)$.

4. Find $f'(4)$ directly from the definition of the derivative (using limits and without l'Hospital's rule):
   $f(x) = \sqrt{x}$

5. Find the limit if it exists. You may use any method (except for a numerical table).

    (a) $\lim\limits_{x \to \infty} \sqrt{9x^2 + x} - 3x$

    (b) $\lim\limits_{x \to \pi^-} \dfrac{\sin(x)}{1 - \cos(x)}$

    (c) $\lim\limits_{x \to 0} \dfrac{x}{\tan^{-1}(4x)}$

6. Differentiate: $F(x) = \displaystyle\int_{2x}^{x^2} e^{t^2}\, dt$

7. Write the definite integral as an appropriate Riemann sum. $\displaystyle\int_{2}^{5} x^2 + 1\, dx$

8. Evaluate the integral, if it exists

    (a) $\displaystyle\int_{1}^{9} \dfrac{\sqrt{u} - 2u^2}{u}\, du$

    (b) $\displaystyle\int 3^x + \dfrac{1}{x} + \sec^2(x)\, dx$

    (c) $\displaystyle\int_{\pi/4}^{\pi/4} \dfrac{t^4 \tan(t)}{2 + \cos(t)}\, dt$

    (d) $\displaystyle\int_{0}^{3} |x^2 - 4|\, dx$

9. A water tank in the shape of an inverted cone with a circular base has a base radius of 2 meters and
   a height of 4 meters. If water is being pumped into the tank at a rate of 2 cubic meters per minute,
   find the rate at which the water level is rising when the water is 3 meters deep. $(V = \frac{1}{3}\pi r^2 h)$

10. Explain why the following is true (if it is): The function $f(x) = \sqrt{1 + 2x}$ can be well approximated by
    $(x + 5)/3$ if x is approximately 4.

11. The graph of $f'(x)$ is shown in the figure. Sketch the graph of f if f is continuous and $f(0) = -1$.
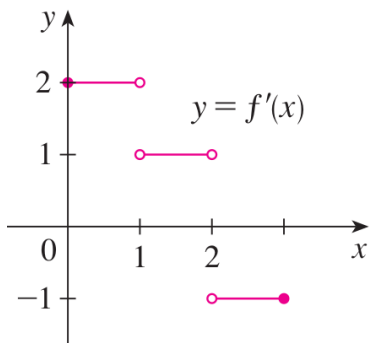
12. Find m and b so that f is continuous and differentiable:

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 2 \\ mx + b & \text{if } x > 2 \end{cases}$$

13. Boat A is traveling north at a constant speed of 10 kilometers per hour. At noon, boat B is located 10 km east of boat A and boat B is traveling east at a constant speed of 10 kilometers per hour. How fast is the distance between the boats increasing at 3:00 PM?

14. Suppose that over a period of 70 years, the population of a country goes from 20 million to 80 million. If the growth is exponential, find the doubling time of the population. Find a formula for the population at any time t.

15. What is the minimum possible surface area of a rectangular box with square base and a volume of 8 cubic feet?

16. If $f(x) = 1/x$ on the interval $[1, 2]$, find the point(s) c that are guaranteed by the Mean Value Theorem.