Sample Final 3

Name:_____

I certify that the work on this exam is my own and that I have not discussed the specific contents of this exam with anyone prior to taking it.

Initials:

Instructions: No calculators are allowed, and no notes beyond what is provided here. Answers with no appropriate justification will receive no credit.

1. Evaluate:

   (a) $\sec(-9\pi/4)$ (b) $\sin^{-1}(\sqrt{3}/2)$ (c) $\log_7(\sqrt{7}/49^3)$

2. True or False, and give a short reason:

   (a) $\frac{d}{dx}(10^x) = x10^{x-1}$

   (b) $\lim_{x \to 1} \dfrac{x^2 + 6x - 7}{x^2 + 5x - 6} = \dfrac{\lim_{x \to 1} x^2 + 6x - 7}{\lim_{x \to 1} x^2 + 5x - 6}$

   (c) If $f(x) = x^2$, then the equation of the normal line at $x = 3$ is: $y - 9 = \frac{-1}{2x}(x - 3)$

3. Differentiate:

   (a) $y = 3^{x^2-1} + (x^2 - 3x + 1)^5$ (b) $y = \dfrac{1 - 2x}{\sqrt[3]{x^5}}$ (c) $y = (x^2 - 1)^{\sin(x)}$

4. Find all vertical and horizontal asymptotes of $f(x) = \dfrac{2x^2 - 2}{x^2 - x - 2}$

5. Derive the formula for the derivative of $y = \sec^{-1}(x)$:

6. Find $f'(1)$ using the definition of the derivative (using limits and you may not use l'Hospital's rule), if $f(x) = \dfrac{x}{x + 1}$

7. Find the limit, if it exists (you may use any method from class):
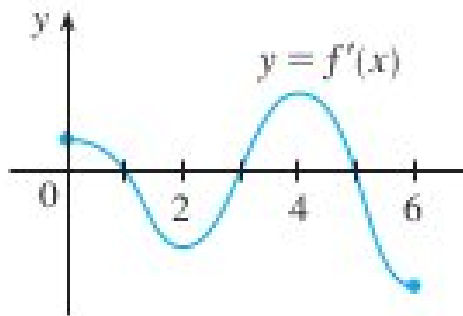
   (a) $\lim_{x \to 3} \dfrac{\sqrt{x + 6} - x}{x^3 - 3x^2}$ (b) $\lim_{x \to -\infty} \dfrac{\sqrt{x^2 - 9}}{2x - 6}$ (c) $\lim_{x \to 0^+} \left(\dfrac{1}{x} - \dfrac{1}{e^x - 1} \right)$

8. Find f if $f''(x) = 6x + \sin(x)$ if $f'(0) = 0$ and $f(0) = 3$.

9. The following is the graph of $f'(x)$:

   (a) On what intervals is f increasing or decreasing?

   (b) On what intervals is f concave up or concave down?

   (c) At what points does f have a local maximum?

   (d) Sketch a graph of f''.

10. A spotlight on the ground shines one a wall 12 meters away. If a man 2 meters tall walks from the spotlight to the wall at a speed of 1.6 meters per second, how fast is the length of the shadow on the building decreasing when he is 4 meters from the building?

11. It has been estimated that since the second half of the 19th century, the population of the United States doubles approximately every 56 years. If the current population is approximately 311 million, when will the population reach half a billion? (Leave your answer in exact form).

12. If $F(x) = \int_0^x 6 - 3x \, dx$, find where the function F is increasing/decreasing.

13. Evaluate:

   (a) $\int_0^1 (x+2)(x+1) \, dx$

   (b) $\int_1^{18} \sqrt{\dfrac{3}{z}} \, dz$

   (c) $\int_0^1 x^e + e^x \, dx$

14. The velocity function for a particle moving along a line is $v(t) = 3t - 5$. (i) Find the displacement on the interval $[0, 3]$, then (ii) find the distance traveled on $[0, 3]$.

15. The shortest distance from an island to a straight shoreline is 5 km. Let P be the point on the shoreline corresponding to this shortest distance. From point P, there is a town 13 kilometers away along the beach. We can travel at 5 k per hour in the water, and 10 k per hour on the beach. Construct a function that can be used to find the point on the beach that will minimize our travel time from the island to the town. You do not have to find the minimum, just construct the function.