Section 3.9: Related Rates

Example 1: Oil Leak- #2, 3.9

An oil rig springs a leak and the oil spreads in a circular patch around the rig. If the radius is increasing at a rate of 30 meters per hour, how fast is the area increasing when the patch has radius 100 meters?

Example 2: Converging Vehicles

Two airplanes approach an airport, one flying due west at 120 MPH, the other due north at 150 MPH. Assuming constant elevation, how fast is the distance between them changing when the westbound plane is 180 miles from the airport, and the northbound plane is 225 miles from the airport.

Other examples

1. A 25 ft ladder is leaning against a vertical wall. It is slipping down the wall at a rate of 1 ft/sec. How fast is the bottom of the ladder slipping away when it is 7 feet from the base of the wall?

2. A street light is mounted at the top of a 20 ft pole. A man 5 feet tall walks away from the pole with a speed of 5 feet per second along a straight path. How fast is the tip of his shadow moving when he is 30 feet from the pole?

3. Sand falls from an overhead bin, accumulating in a conical pile with a radius that is always three times the height. If the sand falls from the bin at a rate of 120 cubic feet per minute, how fast is the height changing when the pile is 10 feet tall? (The volume of a cone is: $V = \frac{1}{3}\pi r^2 h$)

4. An observer stands 200 meters from the launch site of a hot air balloon. The balloon rises vertically at a constant rate of 4 meters per second. How fast is the angle of elevation of the balloon increasing 30 seconds after launch (the angle of elevation is the angle between the ground and the observer's line of sight to the balloon).

5. (Exercise 24) A trough is 10 feet long and its cross section has the shape of an isosceles triangle that is 3 feet across at the top and has a height of 1 ft. If the trough is being filled at a rate of $12 \text{ ft}^3/\text{min}$, how fast is the water level rising when the water is 6 inches deep?

Extra Examples

1. A rocket is shot vertically upward with an initial velocity of 400 ft/sec. Its height after t seconds is

$$s(t) = 400t - 16t^2$$

An observer is standing 1800 feet from the rocket at launch. How fast is the distance from the observer to the rocket changing when the rocket is still rising and is 2400 ft above the ground?