Exam III Review

A Quick Overview:

In this portion of the course, we've been focusing on applications of the derivative. Since Exam 2, we've looked at:

1. Limits:
   (a) L'Hospital's Rule, changing a product or exponent into the right form (a fraction)
   (b) Vertical and Horizontal Asymptotes, Divide by a power of x.
2. Shapes of the graph:
   (a) Higher derivatives
   (b) Increasing/Decreasing, Concave up, concave down.
   (c) Local maxima/minima, Fermat's Theorem, Critical points
   (d) Global maxima/minima, Extreme Value Theorem
3. Optimization
4. Linearization and Differentials

Review Questions

This is not meant to be an all exhaustive list, rather it is meant to give you some random questions out of context. It is important that you understand the homework and the quizzes as well as these exercises. You might also look over the graphing exercises given online as well (given the graphs, figure out which is f, f', f'' and also given the graph of $f'(x)$, find where f is inc/dec and its concavity).

1. Short Answer:
   (a) Give the definition of a **critical point** for a function f:
   (b) State the Extreme Value Theorem:
   (c) State Fermat's Theorem:
   (d) What is the procedure for finding the maximum or minimum of a function $y = f(x)$ on a closed interval, $[a, b]$. How is it that different than if we do not have a closed interval?
   (e) How is finding local extrema different than global?
   (f) What is meant by *linearizing* a function?
2. True or False, and give a short reason:
   (a) $\lim_{x\to\pi^-} \frac{\sin(x)}{1-\cos(x)} = \lim_{x\to\pi^-} \frac{\cos(x)}{\sin(x)} \to -\infty$
   (b) If $f'(a) = 0$, then there is a local maximum or local minimum at $x = a$.

(c) $\lim_{x\to\infty} \sqrt{x^2+2x+4} - x = 0$

(d) There is a vertical asymptote at $x = 2$ for $\frac{\sqrt{x^2+5}-3}{x^2-2x}$

(e) If f has a global minimum at $x = a$, then $f'(a) = 0$.

In the following, "increasing" or "decreasing" will mean for all x:

(f) If $f(x)$ is increasing, and $g(x)$ is increasing, then $f(x) + g(x)$ is increasing.

(g) If $f(x)$ is increasing, and $g(x)$ is increasing, then $f(x)g(x)$ is increasing.

(h) If $f(x)$ is increasing, and $g(x)$ is decreasing, then $f(g(x))$ is decreasing.

3. Maximums and Minimums and related questions.

(a) Find two numbers whose difference is 100 and whose product is a minimum.

(b) A window in the shape of a rectangle for the base is surmounted by a half-circle (see the figure). If the perimeter must be 30 feet, find the dimensions of the window that gives the maximum amount of area (to maximize the amount of light).

(c) Find a positive number such that the sum of the number and its reciprocal is as small as possible.

(d) Find the dimension of the rectangle of largest area that can be drawn if the base of the rectangle is on the $x-$axis and its other vertices are on the parabola $y = 8 - x^2$ (See the figure).

(e) Find the dimensions of the largest rectangle that can be inscribed in a right triangle with legs of lengths 3 and 4, as shown in the Figure.

(f) Find the global maximum and minimum of the given function on the interval provided:

i. $f(x) = \sqrt{9 - x^2}$, $[-1, 2]$

ii. $g(x) = x - 2\cos(x)$, $[-\pi, \pi]$

(g) Find the regions where f is increasing/decreasing:

i. $g(x) = x - 2\sin(x)$ for $0 < x < 3\pi$

ii. $h(x) = \frac{x}{(1+x)^2}$

(h) Set up the expressions to find the maximum area of the rectangle that can be inscribed in a circle of radius 3. Do not solve.

4. Find the local extrema using both the first and second derivative tests.

(a) $y = x^3 - 12x + 1$

(b) $y = x^2 e^x$

(c) $f(x) = x - 2\sin(x)$, $0 < x < 3\pi$

(d) $f(x) = x + \sqrt{1-x}$

5. Find the limit, if it exists.

(a) $\lim_{x\to 0} \dfrac{3^x - 2^x}{x}$

(b) $\lim_{x\to\infty} \dfrac{3x^2 - 5x + 4}{\sqrt{x^4 + 5}}$

(c) $\lim_{x\to\infty} x\tan(1/x)$

(d) $\lim_{x\to\infty} \dfrac{\ln(1+e^x)}{1+x}$

(e) $\lim_{x\to\infty} \left(1+\dfrac{4}{x}\right)^{2x}$

(f) $\lim_{x\to 2^+} \dfrac{x+1}{x^2+x-6}$

6. Compute Δy and dy for the given x and $dx = \Delta x$. Sketch a diagram and label Δx, Δy and dy, if $f(x) = 6 - x^2$, $x = -2$, $\Delta x = 1$.

7. Linearize at $x = 0$:

$$y = \sqrt{x+1}e^{-x^2}$$

Use the linearization to estimate $\sqrt{\frac{3}{2}}e^{-\frac{1}{4}}$

8. A particle moves so that its position at time t is given by $y(t) = t/(1+t^2)$, where t is measured in seconds and y in meters. (a) Find the acceleration at time t. When is it 0? (b) When is the particle speeding up?

9. Find all vertical and horizontal asymptotes for each function:

(a) $f(x) = \dfrac{2x+3}{\sqrt{x^2-2x-3}}$

(b) $f(x) = \sqrt{x+1} - x$

(c) $f(x) = \dfrac{x^2-5x+6}{x-3}$

(d) $f(x) = \dfrac{4x-5}{3x+2}$

10. Estimate by linear approximation the change in the indicated quantity.

(a) The volume, $V = s^3$ of a cube, if its side length s is increased from 5 inches to 5.1 inches.

(b) The volume, $V = \frac{4}{3}\pi r^3$ of a sphere, if its radius changes from 2 to 2.1

(c) The volume, $V = \frac{1000}{p}$, of a gas, if the pressure p is decreased from 100 to 99.

(d) The period of oscillation, $T = 2\pi\sqrt{\frac{L}{32}}$, of a pendulum, if its length L is increased from 2 to 2.2.