Quiz 9 solutions

1. Given the function $g(x) = 200 + 8x^3 + x^4$:

   (a) Find the intervals of increase or decrease.
   SOLUTION:

   $$g'(x) = 24x^2 + 4x^3 = 0 \quad \Rightarrow \quad g'(x) = 4x^2(6 + x)$$

   The sign of $g'(x)$ will be the sign of $x + 6$, so if $x < -6$, g is decreasing, and if $-6 < x < 0$ and $x > 0$, then g is increasing.

   (b) Find the local maximum and local minimum values.
   SOLUTION: From our analysis above, there is a local (and global) minimum at $x = -6$ (where $f(-6) = -232$). There is no local maximum

   (c) Find the intervals of concavity and the inflection points.
   SOLUTION: Take the second derivative:

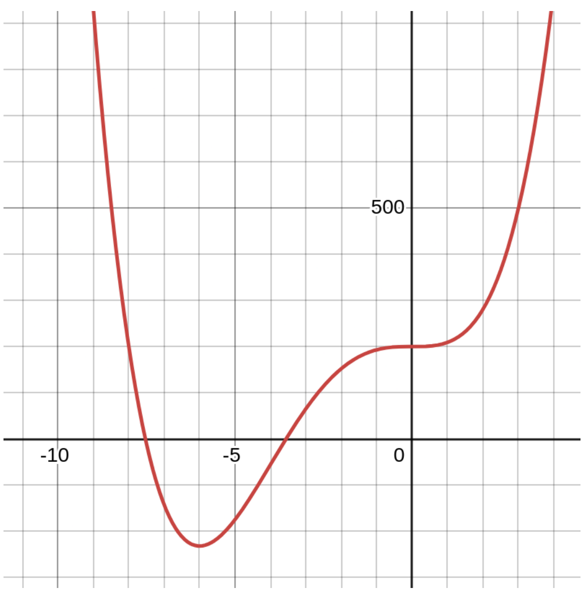
   $$g''(x) = 48x + 12x^2 = 12x(4 + x)$$

   Using a sign chart (it's a parabola so we'll do it all at once):

$12x(4+x)$	$+$	$-$	$+$
	$x < -4$	$-4 < x < 0$	$x > 0$

   The function is concave up when $x < -4$, or $x > 0$, and is concave down in between.

For verification, here's the graph:



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2. Compute each limit, if it exists.

   (a) $\lim_{x\to\infty} x^3 e^{-2x}$

   SOLUTION: We can apply l'Hospital's rule several times:

$$\lim_{x\to\infty} \frac{x^3}{e^{2x}} = \lim_{x\to\infty} \frac{3x^2}{2e^{2x}} = \lim_{x\to\infty} \frac{6x}{4e^{2x}} = \lim_{x\to\infty} \frac{6}{8e^{2x}} = 0$$

   (b) $\lim_{x\to\infty} x^{1/x}$

   One way to think about this is that, since $A = e^{\ln(A)}$, then

$$x^{1/x} = e^{\ln(x^{1/x})} = e^{\ln(x)/x}$$

   Therefore, to compute the limit:

$$\lim_{x\to\infty} e^{\ln(x)/x} = e^{\lim_{x\to\infty} \ln(x)/x}$$

   Therefore, we just need to compute that exponent using l'Hospital's rule:

$$\lim_{x\to\infty} \frac{\ln(x)}{x} = \lim_{x\to\infty} \frac{1/x}{1} = 0$$

   And going back, the full limit is then $e^0 = 1$.