Review Questions, Exam 3

You should also look over the homework, quizzes and the related rates handout.

1. A child is flying a kite. If the kite is 90 feet above the child's hand level and the wind is blowing it on a horizontal course at 5 feet per second, how fast is the child paying out cord when 150 feet of cord is out? (Assume that the cord forms a line- actually an unrealistic assumption).

2. Use differentials to approximate the increase in area of a soap bubble, when its radius increases from 3 inches to 3.025 inches ($A = 4\pi r^2$)

3. True or False, and give a short reason:

   (a) If air is being pumped into a spherical rubber balloon at a constant rate, then the radius will increase, but at a slower and slower rate.

   (b) If $y = x^5$, then $dy \geq 0$

   (c) If a car *averages* 60 miles per hour over an interval of time, then at some instant, the speedometer must have read exactly 60.

   (d) A global maximum is always a local maximum.

   (e) The linear function $f(x) = ax + b$, where a, b are constant, and $a \neq 0$, has no minimum value on any open interval. (An interval is open if it does not include its endpoints).

   (f) Suppose P and Q are two points on the surface of the sea, with Q lying generally to the east of P. It is possible to sail from P to Q (always sailing roughly east), without *ever* sailing in the exact direction from P to Q.

   (g) If $f(x) = 0$ has three (distinct) real solutions, then $f'(x) = 0$ must have two solutions (Assume f is differentiable).

4. Linearize at $x = 0$:
$$y = \sqrt{x}e^{x^2}(x^2 + 1)^{10}$$

5. Estimate by linear approximation the change in the indicated quantity.

   (a) The volume, $V = s^3$ of a cube, if its side length s is increased from 5 inches to 5.1 inches.

   (b) The volume, $V = \frac{1000}{p}$, of a gas, if the pressure p is decreased from 100 to 99.

   (c) The period of oscillation, $T = 2\pi\sqrt{\frac{L}{32}}$, of a pendulum, if its length L is increased from 2 to 2.2.

6. For the following problems, find where f is increasing or decreasing. If asked, also check concavity.

   (a) $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ (Also check for concave up/down)

   (b) $f(x) = \frac{x}{x+1}$

   (c) $f(x) = x\sqrt{x^2 + 1}$

7. Show that the given function satisfies the hypotheses of the Mean Value Theorem. Find all numbers c in that interval that satisfy the conclusion of that theorem. For comparison purposes, given these functions and intervals, what would the Intermediate Value Theorem conclude? Finally, find the global max and global min for each function.

   (a) $f(x) = x^3$, $[-1, 1]$

   (b) $f(x) = \sqrt{x - 1}$, $[2, 5]$

   (c) $f(x) = x + \frac{1}{x}$, $[1, 5]$

8. Show that $f(x) = x^{2/3}$ does not satisfy the hypotheses of the mean value theorem on $[-1, 27]$, but nevertheless, there is a c for which:
$$f'(c) = \frac{f(27) - f(-1)}{27 - (-1)}$$
   Find the value of c.

9. At 1:00 PM, a truck driver picked up a fare card at the entrance of a tollway. At 2:15 PM, the trucker pulled up to a toll booth 100 miles down the road. After computing the trucker's fare, the toll booth operator summoned a highway patrol officer who issued a speeding ticket to the trucker. (The speed limit on the tollway is 65 MPH).

   (a) The trucker claimed that she hadn't been speeding. Is this possible? Explain.

   (b) The fine for speeding is $35.00 plus $2.00 for each mph by which the speed limit is exceeded. What is the trucker's minimum fine?

10. Let $f(x) = \frac{1}{x}$

   (a) What does the Extreme Value Theorem (EVT) say about f on the interval $[0.1, 1]$?

   (b) Although f is continuous on $[1, \infty)$, it has no minimum value on this interval. Why doesn't this contradict the EVT?

11. Let f be a function so that $f(0) = 0$ and $\frac{1}{2} \leq f'(x) \leq 1$ for all x. Use the Mean Value Theorem to explain why $f(2)$ cannot be 3.

12. Sketch the graph of a function that satisfies all of the given properties:

$$f'(-1) = 0, f'(1) \text{ does not exist}, f'(x) < 0 \text{ if } |x| < 1, f'(x) > 0 \text{ if } |x| > 1$$

$$f(-1) = 4, f(1) = 0, f''(x) > 0 \text{ if } x > 0$$

13. Use Newton's Method to compute x_1, x_2, x_3, if

$$f(x) = x^3 - x^2 - 1$$

   and $x_0 = 1$.

14. Problem 92, p. 270

15. Be sure you understand Problem 1, Section 4.9

16. Find the local maximums and local minimums of f using both the first and second derivative tests:

$$f(x) = x + \sqrt{1 - x}$$

17. Find a cubic function $y = ax^3 + bx^2 + cx + d$ that has a local maximum value of 3 at $x = -2$ and a local minimum value of 0 at $x = 1$.

18. A metal storage tank is to hold a certain volume, V. It is to be constructed in the shape of a right circular cylinder surmounted by a hemisphere. What dimensions will require the least amount of metal? (Note: There is a bottom to the tank, and you should consider V to be some given constant- so your answer might depend on V).

19. If you want more practice with some related rates problems, see the handout on our Calculus web page.