**Calculus I Review Questions**

**NOTE: You should also look over your old exams, old review sheets, old quizzes and homework.**

1. Compare and contrast the three "Value Theorems" of the course. When you would typically use each.

2. List the three things we need to check to see if a function $f$ is continuous at $x = a$.

3. Derive the formula for the derivative of $y = \sec^{-1}(x)$.

4. Find the point on the parabola $x + y^2 = 0$ that is closest to the point $(0, -3)$.

5. Write the equation of the line tangent to $x = \sin(2y)$ at $x = 1$.

6. For what values of $A, B, C$ will $y = Ax^2 + Bx + C$ satisfy the differential equation:
$$\frac{1}{2}y'' - 2y' + y = 3x^2 + 2x + 1$$

7. Compute the derivative of $y$ with respect to $x$:
   (a) $y = \sqrt[3]{2x+1}\sqrt[5]{3x-2}$
   (b) $y = \frac{1}{1+u^2}$, where $u = \frac{1}{1+x^2}$
   (c) $\sqrt[3]{y} + \sqrt[3]{x} = 4xy$
   (d) $\sqrt{x+y} = \sqrt[3]{x-y}$
   (e) $y = \sin(2\cos(3x))$
   (f) $y = (\cos(x))^{2x}$
   (g) $y = (\tan^{-1}(x))^{-1}$
   (h) $y = \sin^{-1}(\cos^{-1}(x))$
   (i) $y = \log_{10}(x^2 - x)$
   (j) $y = x^{x^2+2}$
   (k) $y = e^{\cos(x)} + \sin(5^x)$
   (l) $y = \cot(3x^2 + 5)$
   (m) $y = \sqrt{\sin(\sqrt{x})}$
   (n) $\sqrt{x} + \sqrt[3]{y} = 1$
   (o) $x\tan(y) = y - 1$
   (p) $y = \frac{-2}{\sqrt[4]{t^3}}$, where $t = \ln(x^2)$.
   (q) $y = x3^{-1/x}$

8. Let $f(x) = x2^{x+1}$. Without explicitly computing the inverse, what is the equation of the tangent line to $f^{-1}(x)$ at $x = 4$? HINT: The point $(1, 4)$ goes through the graph of $f$.

9. Find the local maximums and minimums: $f(x) = x^3 - 3x + 1$ Show your answer is correct by using both the first derivative test and the second derivative test.

10. Compute the limit, if it exists. You may use any method (except a numerical table).
    (a) $\lim_{x\to 0} \frac{x - \sin(x)}{x^3}$
    (b) $\lim_{x\to 0} \frac{1 - e^{-2x}}{\sec(x)}$
    (c) $\lim_{x\to 4^+} \frac{x-4}{|x-4|}$
    (d) $\lim_{x\to -\infty} \sqrt{\frac{2x^2-1}{x+8x^2}}$
    (e) $\lim_{x\to\infty} \sqrt{x^2+x+1} - \sqrt{x^2-x}$
    (f) $\lim_{h\to 0} \frac{(1+h)^{-2} - 1}{h}$
    (g) $\lim_{x\to\infty} x^3 e^{-x^2}$
    (h) $\lim_{x\to 1} \frac{x^{1000} - 1}{x - 1}$
    (i) $\lim_{x\to 0} \frac{x}{\tan^{-1}(4x)}$
    (j) $\lim_{x\to 1} x^{\frac{1}{1-x}}$

11. Determine all vertical/horizontal asymptotes and critical points of $f(x) = \frac{2x^2}{x^2-x-2}$

12. Find values of $m$ and $b$ so that (1) $f$ is continuous, and (2) $f$ is differentiable.
$$f(x) = \begin{cases} x^2 & \text{if } x \le 2 \\ mx + b & \text{if } x > 2 \end{cases}$$

13. Find the local and global extreme values of $f(x) = \frac{x}{x^2+x+1}$ on the interval $[-2, 0]$.

14. Suppose $f$ is differentiable so that:
$$f(1) = 1,\ f(2) = 2,\ f'(1) = 1\ f'(2) = 2$$
If $g(x) = f(x^3 + f(x^2))$, evaluate $g'(1)$.

15. Let $x^2y + a^2xy + \lambda y^2 = 0$
    (a) Let $a$ and $\lambda$ be constants, and let $y$ be a function of $x$. Calculate $\frac{dy}{dx}$:
    (b) Let $x$ and $y$ be constants, and let $a$ be a function of $\lambda$. Calculate $\frac{da}{d\lambda}$:

16. Show that $x^4 + 4x + c = 0$ has at most one solution in the interval $[-1, 1]$.

17. True or False, and give a short explanation.

    (a) If $f$ has an absolute minimum at $c$, then $f'(c) = 0$.

    (b) If $f$ is differentiable, then

    $$\frac{d}{dx}\sqrt{f(x)} = \frac{f'(x)}{2\sqrt{f(x)}}$$

    (c) $\frac{d}{dx}(10^x) = x10^{x-1}$

    (d) If $f'(x)$ exists and is nonzero for all $x$, then $f(1) \neq f(0)$.

    (e) If $y = ax + b$, then $\frac{dy}{da} = x$

    (f) If $2x + 1 \leq f(x) \leq x^2 + 2$ for all $x$, then $\lim_{x\to 1} f(x) = 3$.

    (g) If $f'(r)$ exists, then

    $$\lim_{x\to r} f(x) = f(r)$$

    (h) If $f$ and $g$ are differentiable, then:

    $$\frac{d}{\mathrm{dx}}(f(g(x)) = f'(x)g'(x)$$

    (i) If $f(x) = x^2$, then the equation of the tangent line at $x = 3$ is: $y - 9 = 2x(x - 3)$

    (j) $\displaystyle\lim_{\theta\to\frac{\pi}{3}} \frac{\cos(\theta) - \frac{1}{2}}{\theta - \frac{\pi}{3}} = -\sin\left(\frac{\pi}{3}\right)$

    (k) There is no solution to $\mathrm{e}^x = 0$

    (l) $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right) = \frac{2\pi}{3}$

    (m) $5^{\log_5(2x)} = 2x$, for $x > 0$.

    (n) $\frac{d}{dx}\ln(|x|) = \frac{1}{x}$, for all $x \neq 0$.

    (o) $\frac{d}{dx}10^x = x10^{x-1}$

    (p) If $x > 0$, then $(\ln(x))^6 = 6\ln(x)$

    (q) The most general antiderivative of $x^{-2}$ is $\frac{-1}{x} + C$.

18. Find the domain of $\ln(x - x^2)$:

19. Find the value of $c$ guaranteed by the Mean Value Theorem, if $f(x) = \frac{x}{x+2}$ on the interval $[1, 4]$.

20. Given that the graph of $f$ passes through the point $(1, 6)$ and the slope of the tangent line at $(x, f(x))$ is $2x + 1$, find $f(2)$.

21. A fly is crawling from left to right along the curve $y = 8 - x^2$, and a spider is sitting at $(4, 0)$. At what point along the curve does the spider first see the fly?

22. Compute the limit, without using L'Hospital's Rule. $\displaystyle\lim_{x\to 7} \frac{\sqrt{x+2} - 3}{x - 7}$

23. For what value(s) of $c$ does $f(x) = cx^4 - 2x^2 + 1$ have both a local maximum and a local minimum?

24. If $f(x) = \sqrt{1 - 2x}$, determine $f'(x)$ *by using the definition of the derivative.*

25. A *point of inflection* for a function $f$ is the $x$ value for which $f''(x)$ changes sign (either from positive to negative or vice versa).

    Find constants $a$ and $b$ so that $(1, 6)$ is an inflection point for $y = x^3 + ax^2 + bx + 1$.

    Hint: The IVT might come in handy

26. Suppose that $F(x) = f(g(x))$ and $g(3) = 6$, $g'(3) = 4$, $f(3) = 2$ and $f'(6) = 7$. Find $F'(3)$.

27. Find the dimensions of the rectangle of largest area that has its base on the $x-$axis and the other two vertices on the parabola $y = 8 - x^2$.

28. Let $G(x) = h(\sqrt{x})$. Then where is $G$ differentiable? Find $G'(x)$.

29. If position is given by: $f(t) = t^4 - 2t^3 + 2$, find the times when the acceleration is zero. Then compute the velocity at these times.

30. If $y = \sqrt{5t - 1}$, compute $y'''$.

31. Find a second degree polynomial so that $P(2) = 5$, $P'(2) = 3$, and $P''(2) = 2$.

32. Find a function $f(x)$ so that $f'(x) = 4 - 3(1 + x^2)^{-1}$, and $f(1) = 0$

33. If $f(x) = (2 - 3x)^{-1/2}$, find $f(0)$, $f'(0)$, $f''(0)$.

34. Car A is traveling west at 50 mi/h, and car B is traveling north at 60 mi/h. Both are headed for the intersection between the two roads. At what rate are the cars approaching each other when car A is 0.3 mi and car B is 0.4 mi from the intersection?

35. Compute $\Delta y$ and $dy$ for the value of $x$ and $\Delta x$: $f(x) = 6 - x^2$, $x = -2$, $\Delta x = 0.4$.

36. Find the linearization of $f(x) = \sqrt{1 - x}$ at $x = 0$.

37. Find $f(x)$, if $f''(x) = t + \sqrt{t}$, and $f(1) = 1$, $f'(1) = 2$.

38. Find $f'(x)$ directly from the definition of the derivative (using limits and without L'Hospital's rule):

(a) $f(x) = \sqrt{3-5x}$
(b) $f(x) = x^2$
(c) $f(x) = x^{-1}$

39. If $f(0) = 0$, and $f'(0) = 2$, find the derivative of $f(f(f(f(x))))$ at $x = 0$.

40. Differentiate:

$$f(x) = \begin{cases} \sqrt{x} & \text{if } x \geq 0 \\ \sqrt{-x} & \text{if } x < 0 \end{cases}$$

Is $f$ differentiable at $x = 0$? Explain.

41. $f(x) = |\ln(x)|$. Find $f'(x)$.

42. $f(x) = xe^{g(\sqrt{x})}$. Find $f'(x)$.

43. Find a formula for dy/dx: $x^2 + xy + y^3 = 0$.

44. Show that 5 is a critical number of $g(x) = 2+(x-5)^3$, but that $g$ does not have a local extremum there.

45. Find the general antiderivative:

(a) $f(x) = 4 - x^2 + 3e^x$
(b) $f(x) = \frac{3}{x^2} + \frac{2}{x} + 1$
(c) $f(x) = \frac{1+x}{\sqrt{x}}$

46. Find the slope of the tangent line to the following at the point (3,4): $x^2 + \sqrt{y}x + y^2 = 31$

47. Find the critical values: $f(x) = |x^2 - x|$

48. Does there exist a function $f$ so that $f(0) = -1$, $f(2) = 4$, and $f'(x) \leq 2$ for all $x$?

49. Find a function $f$ so that $f'(x) = x^3$ and $x+y = 0$ is tangent to the graph of $f$.

50. Find $dy$ if $y = \sqrt{1-x}$ and evaluate $dy$ if $x = 0$ and $dx = 0.02$. Compare your answer to $\Delta y$

51. Fill in the question marks: If $f''$ is positive on an interval, then $f'$ is ? and $f$ is ?.

52. If $f(x) = x - \cos(x)$, $x$ is in $[0, 2\pi]$, then find the value(s) of $x$ for which

(a) $f(x)$ is greatest and least.
(b) $f(x)$ is increasing most rapidly.
(c) The slopes of the lines tangent to the graph of $f$ are increasing most rapidly.

53. Show there is *exactly* one solution to: $\ln(x) = 3 - x$.

54. Approximate the change in volume of a cone, if we assume the height to be constant and $r$ changes from 2 to 2.1. ($V = \frac{1}{3}\pi r^2 h$)

55. Sketch the graph of a function that satisfies all of the given conditions:

$$f(1) = 5 \qquad f(4) = 2 \qquad f'(1) = f'(4) = 0$$
$$\lim_{x\to 2^+} f(x) = \infty, \qquad \lim_{x \to 2^-} f(x) = 3 \qquad f(2) = 4$$

56. If $s^2 t + t^3 = 1$, find $\frac{dt}{ds}$ and $\frac{ds}{dt}$.

57. Find the specific antiderivative:

(a) $f'(x) = 3\sqrt{x} - \frac{1}{\sqrt{x}}, \quad f(1) = 2$
(b) $f''(x) = x^2 + 3\cos(x), \quad f(0) = 2, f'(0) = 3$
(c) $f''(x) = 3e^x + 5\sin(x), \quad f(0) = 1, f'(0) = 2$
(d) $f'(x) = \frac{4}{\sqrt{1-x^2}}, \quad f(1/2) = 1.$