Quick Sheet: Intro to the Derivative

1. Definition:

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

   You should choose the version that will make it the easiest to do the necessary algebra to simplify/cancel/take the limit.

2. Computing the derivative: Factor and cancel- This is all the algebra that we've used previously to take limits. Nothing new here, except that we first have to figure out what the expressions for either:

$$\frac{f(a+h) - f(a)}{h} \quad \text{or} \quad \frac{f(x) - f(a)}{x - a}$$

   look like.

3. Interpretations of the derivative:
   - $f'(a)$ is the slope of the tangent line at to $f(x)$ at $x = a$.
   - If f is distance, $f'(a)$ is the instantaneous velocity at $x = a$.
   - If f is a cost function, $f'(a)$ is the marginal cost.
   - $f'(a)$ is the "local multiplier" (or instantaneous rate of change) for f at $x = a$.
   - We can view $f'(x)$ to be the derivative *function* associated with $f(x)$. That is, the range of $f'(x)$ tells us the value of the slope of the tangent line for each x in its domain.
   - Other notation for the derivative:

$$f'(x) = y' = \frac{dy}{dx} = \frac{d}{dx} f$$

4. Definitions and Theorems about differentiability.
   - f is said to be differentiable at $x = a$ if $f'(a)$ exists (this is a local definition).
   - f is said to be differentiable on an interval (a, b) if it is differentiable at every x in the interval (this is a global definition).
   - All differentiable functions are continuous. (NOT all continuous functions are differentiable! Template: $y = |x|$)
   - Because of the previous theorem, differentiability is a stronger condition than continuity. Intuitively, to be differentiable, the function not only must be continuous, but also be *smooth*- The graph of f cannot have any sharp corners.
   - Because of the previous property, we think of *differentiable* functions as being "locally linear"- they are locally very well approximated by their tangent lines.

5. Side Remark: Is it possible for a function to be everywhere continuous, but not differentiable anywhere? Yes- Many examples are known in the context of *fractal geometry.* In these cases, the graph of f is incredibly "crinkled up" no matter how far one zooms into its graph! (A template example: The Koch Curve).

6. Graphical Skills:
   - Be able to tell if a function is differentiable by looking at its graph. (See the textbook for examples).
   - Be sure you can sketch the graph of the derivative function given the graph of a function (see the textbook for examples).