Quick Sheet: Horizontal Asymptotes

1. In this worksheet, we examine how to compute

$$\lim_{x \to \pm\infty} f(x)$$

   If such a limit exists (call it L), then the line $y = L$ is said to be a horizontal asymptote of f.

2. Unlike vertical asymptotes, it is possible for f to cross a horizontal asymptote.

3. A main template: Let $r > 0$. Then

$$\lim_{x \to \infty} \frac{1}{x^r} = 0 \tag{1}$$

   - We have to be a little more careful for negative values of x. That is, as long as x^r is *defined*, then we get the same statement as above for $x \to -\infty$.
   - The following is a useful fact for algebraic manipulation:

     If x is negative, then $x = -\sqrt{x^2}$

4. Computation of limits at infinity:

   - When given a quotient, divide numerator and denominator to take advantage of Equation 1.
   - A difference of functions: "Rationalize" to make it a fraction, then use the previous idea.
   - Templates:
     $y = \tan^{-1}(x)$ has two horizontal asymptotes, $y = \frac{\pi}{2}$ and $y = \frac{-\pi}{2}$.
     $y = e^x$ has $y = 0$ as a horizontal asymptote (for $x \to -\infty$).

5. Some general rules to remember:

   - If each item goes to infinity, so does its sum. The same CANNOT be said about a difference! Use the idea given previously when taking the limit of a difference.
   - If each item goes to infinity, so does the product. The same CANNOT be said about a quotient.

6. A general rule about the infinite limit of a rational function, $p(x)/q(x)$.

   - If the degree of $p >$ degree of q, the limit is infinite.
   - If the degrees are the same, the limit is the ratio of the leading terms.
   - If the degree of $p <$ degree of q, the limit is zero.