Quick Sheet: Limits

1. Definition: If, as $x \to a$, $f(x) \to L$, we say that the limit as x approaches a exists, and write:

$$\lim_{x \to a} f(x) = L$$

2. You should also know what the following mean:

$$\lim_{x \to a^-} f(x) \text{ and } \lim_{x \to a^+} f(x) \text{ and } \lim_{x \to \pm\infty} f(x) \text{ and } \lim_{x \to a} f(x) = \pm\infty$$

   - The first and second limits are the "left-" and "right-" handed limits.
   - The third expression is analyzed in the section on Horizontal Asymptotes.
   - The last expression is the definition of a vertical asymptote.

3. The limit of a sum, difference, product or quotient is the sum, difference, product or quotient of the limits, as long as each limit exists separately, AND, in the case of a quotient, the limit of the denominator is not zero.

4. We also have:

$$\lim_{x \to a} x^n = a^n, \quad \text{and} \quad \lim_{x \to a} c = c$$

$$\lim_{x \to a} (f(x))^n = \left(\lim_{x \to a} f(x)\right)^n \quad \text{and} \quad \lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)}$$

5. To formally compute a limit, we use the rules given above.

6. To compute a limit given an expression for $f(x)$, there are a few simple tools to remember:

   - Factor and cancel, if possible.
   - If it looks like it might help, "rationalize" the fraction.
   - For limits at infinity, see the horizontal asymptote worksheet.

7. NOTE: Also be able to compute limits graphically. See the Calculus textbook for these types of problems- I have not included them here.

8. The Squeeze Theorem: If we can find functions $g(x)$ and $h(x)$ so that, for a given f, the following statements are true:

$$g(x) \le f(x) \le h(x) \text{ and } \lim_{x \to a} g(x) = \lim_{x \to a} h(x) = L$$

   then $\lim_{x \to a} f(x) = L$ also. Normally, we try to find simple functions g and h for the more complicated f.

   Template example: $-x^2 \le x^2 \sin\left(\frac{1}{x}\right) \le x^2$