REVIEW QUESTIONS, Calculus I
Fall 2003
Sections 1.1-1.11

The following questions are meant to be a supplement to, and not a replacement for, the homework and quizzes. Be sure that all your questions on those items have been answered!

1. True or False, and explain:
   (a) The derivative of a polynomial is a polynomial.
   (b) If f is continuous and $f(1) = 3$, $f(2) = 4$, then there is an r so that $f(r) = \pi$.
   (c) If f is differentiable and positive, then $\frac{d}{dx}\left(\sqrt{f(x)}\right) = \frac{f'(x)}{2\sqrt{f(x)}}$
   (d) The equation of the tangent line to $y = x^3$ at $x = 1$ is:
   $$y - 1 = 2x^2(x - 1)$$
   (e) All functions are continuous on their domains.
   (f) If $\lim_{h\to 0} \frac{f(3+h) - f(3)}{h} = 5$, then f is continuous at $x = 3$.
   (g) If, when taking a limit of a rational function, we get $\frac{0}{0}$, then the limit does not exist.
   (h) $f(x) = |x|$ is not continuous at $x = 0$.
   (i) If $y = ax + b$, then $\frac{dy}{da} = x$
   (j) If $f(x) = (3x^2+6x)^4$, then $f'(x) = 4(6x+6)^3$
2. Definitions: Finish each of the following definitions
   (a) A function f is continuous at $x = a$ if:
   (b) The derivative of f at $x = a$ is:
3. How would we use the product rule and chain rule to differentiate:
   $$h(x) = \frac{f(x)}{g(x)}$$
4. The definition of continuity actually means that we have to check three things. What are they?

   Graphically, give examples of functions that satisfy only one of the three things. Graphically give examples of functions that satisfy only two of the three things.
5. Finish the statement of the Intermediate Value Theorem (IVT): Let f be continuous on $[a, b]$, and let ν be any point between $f(a)$ and...

   Graphically give an example of a discontinuous function that does not satisfy the IVT (show which values of ν do not work).
6. How is the IVT normally used (what kinds of questions did we ask in the homework?)

   How would you use the IVT to show that an expression of the form $f(x) = g(x)$ has a solution?
7. Give (graphically) an example of a function that has all of the following features:
   $$\lim_{x\to 3^-} f(x) = 2,\ \lim_{x\to 3^+} f(x) = -1, f(3) = 0$$
   and f is continuous, but not differentiable, at $x = 5$.
8. Each limit is the derivative of some function f at some number a. State the function f and the value of a:
   (a) $\lim_{h\to 0} \frac{\sqrt{1+h} - 1}{h}$
   (b) $\lim_{x\to 1} \frac{\sqrt[9]{x} - 1}{x - 1}$
   (c) $\lim_{h\to 0} \frac{\frac{1}{3+h} - \frac{1}{3}}{h}$
9. A space traveler is moving from left to right along the curve $y = x^2$. When she shuts off the engines, she will go off along the tangent line at that point. At what point should she shut off the engines in order to reach the point $(4, 15)$?
10. Find the coordinates of the point on the curve $y = (x-2)^2$ at which the tangent line is perpendicular to the line $2x - y + 2 = 0$.
11. If $V = 3w^2 + \frac{5}{w}$, $w = \sqrt{u+1}$, $u = t^2 + t$, then compute the rate of change of V with respect to w, then with respect to u, then with respect to t.
12. Show that there must be at least one real solution to:
    $$x^5 = x^2 + 4$$
13. Find the equation of the tangent line to the following curve at the given point:
    (a) $f(x) = \frac{x^2+2x+5}{x}$ at $x = 1$
    (b) $f(x) = (x^2+3x)^5 + 2(x^2+3x)^4 + 3$ at $x = 0$.
14. For the function $f(x) = x^2$, find and simplify the expression
    $$\frac{f(x+h) - f(x-2h)}{3h}$$

15. Find all values of x for which f is continuous:

$$\text{(i) } f(x) = \sqrt{x^2 - 4x}, \qquad \text{(ii) } f(x) = \sqrt{\frac{x^2 - 4}{1 - x^2}}$$

16. Rewrite the function as a piecewise defined function (which gets rid of the absolute value signs):

$$f(x) = \frac{|3x + 2|}{3x + 2} \qquad f(x) = \left| \frac{x - 2}{(x + 1)(x + 2)} \right|$$

17. Compute each limit algebraically (if it exists):

(a) $\lim_{h \to 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h}$

(b) $\lim_{x \to 2} \frac{x^2 - 4}{\frac{1}{x} - \frac{1}{2}}$

(c) $\lim_{x \to 2} \frac{x^2 - 2x - 3}{x - 3}$

(d) $\lim_{x \to 3} \frac{1}{(x - 3)}$

(e) $\lim_{x \to 0} x^2 \sin\left(\frac{x^2 - 5x}{x^{26}}\right)$

(f) $\lim_{x \to 2} \frac{x^2 - 4}{2x^2 + 3x - 14}$

(g) $\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{|x|}\right)$

18. Find all the values of a for which f will be continuous for all real values.

(a) $f(x) = \begin{cases} 4 - x^2 & \text{if } x \leq 3 \\ 3 - ax & \text{if } x > 3 \end{cases}$

(b) $f(x) = \begin{cases} x^2 - 2 & \text{if } x \leq a \\ 2x - 1 & \text{if } x > a \end{cases}$

(c) $f(x) = \begin{cases} \frac{x^2 - 16}{x^2 - 4} & \text{if } x \neq \pm 2 \\ a & \text{if } x = \pm 2 \end{cases}$

19. Find all value(s) of a so that $\lim_{x \to 3} \frac{x^3 - 2x^2 + 2x + a}{x^2 - 3x}$, then compute the limit(s).

20. Compute $f'(x)$ if $f(x) = \frac{x}{x+1}$ by using the definition.

21. Compute $h'(w)$ is $h(w) = \frac{3}{w}$ by using the definition.

22. Compute the velocity if the position is given by $f(t) = \sqrt{t}$ by using the definition of the derivative.

23. Compute the derivative using our shortcut rules:

(a) $y(x) = \frac{5}{\sqrt[5]{x^6}}$

(b) $f(x) = 3x^2 - 5\sqrt[3]{x} - \frac{1}{3x}$

(c) $h(x) = (3x^2 + 15x - 9)^{4/3}$

(d) $g(s) = \frac{2}{\sqrt{2 - 3s}}$

(e) $h(t) = \frac{3t - 5}{t^2 - 6t}$

(f) $f(u) = (3x + 5)\sqrt{2x + 1}$

(g) $p(t) = x^2(2x - 7)^3$