Exam II Review Questions

In this portion of the course, we discussed finding the maximum and minimum of functions, either on a closed interval or not, and we reviewed trigonometric functions, the derivatives of trigonometric functions, the inverse trigonometric functions, and their inverses.

1. Short Answer:
   (a) State the Extreme Value Theorem:
   (b) What is the restriction on the domain of the sine function so that it is invertible? On the tangent function? On the cosine function?
   (c) What is the procedure for finding the maximum or minimum of a function $y = f(x)$ on a closed interval, $[a, b]$.
   (d) What is the procedure for finding a local maximum or minimum?
   (e) What is the procedure for finding a global maximum or minimum (assume the domain is all reals)?

2. True or False, and give a short reason:
   (a) If $f'(a) = 0$, then there is a local maximum or local minimum at $x = a$.
   (b) $\sin^{-1}(x) = \dfrac{1}{\sin(x)}$
   (c) $\tan(\tan^{-1}(x)) = x$ for all x
   In the following, "increasing" or "decreasing" will mean for all x:
   (d) If $f(x)$ is increasing, and $g(x)$ is increasing, then $f(x) + g(x)$ is increasing.
   (e) If $f(x)$ is increasing, and $g(x)$ is increasing, then $f(x)g(x)$ is increasing.
   (f) If $f(x)$ is increasing, and $g(x)$ is decreasing, then $f(g(x))$ is decreasing.

3. Maximums and Minimums and related questions.
   (a) Find two numbers whose difference is 100 and whose product is a minimum.
   (b) A window in the shape of a rectangle for the base is surmounted by a half-circle (see the figure). If the perimeter must be 30 feet, find the dimensions of the window that gives the maximum amount of area (to maximize the amount of light).
   (c) Find a positive number such that the sum of the number and its reciprocal is as small as possible.
   (d) Find the dimension of the rectangle of largest area that can be drawn if the base of the rectangle is on the $x-$axis and its other vertices are on the parabola $y = 8 - x^2$ (See the figure).
   (e) Find the dimensions of the largest rectangle that can be inscribed in a right triangle with legs of lengths 3 and 4, as shown in the Figure.
   (f) Find the point on the (sideways) parabola $y^2 = 2x$ closest to the point $(1, 4)$. Hint: It suffices to find the minimum of the distance squared.
   (g) Find the global maximum and minimum of the given function on the interval provided:
      i. $f(x) = \sqrt{9 - x^2}$, $[-1, 2]$
      ii. $g(x) = x - 2\cos(x)$, $[-\pi, \pi]$
      iii. $h(x) = x^2 + \frac{2}{x}$, $\left[\frac{1}{2}, 2\right]$
   (h) Find the regions where f is increasing/decreasing:
      i. $f(x) = x^3 - 12x + 1$
      ii. $g(x) = x - 2\sin(x)$ for $0 < x < 3\pi$

iii. $h(x) = \frac{x}{(1+x)^2}$

(i) Set up the expressions to find the maximum area of the rectangle that can be inscribed in a circle of radius 3. Do not solve.

4. Differentiate:

(a) $y = \sin(3x)$

(b) $y = \arctan(x)$

(c) $y = \sqrt{3x} - \cos(x^2)$

(d) $y = \sin(2x)\cos(3x)$

(e) $f(x) = \arcsin(1 - 3x)$

(f) $f(x) = \tan(x^2 - 3x + 4)$

(g) $y = \sec(x)$

(h) $y = \sin^2(3x - 5)$

(i) $y = \arctan(\arcsin(\sqrt{x}))$

5. Other Trig Questions:

(a) Find the remaining trig ratios, if:

i. $\tan(\alpha) = 2$, α in 1st Quadrant

ii. $\sec(\phi) = -3/2$, ϕ is in the 2d Quadrant

iii. $\cot(\beta) = 3$, $\pi < \beta < 2\pi$

(b) Find all values in $[0, 2\pi]$ that satisfy the inequality:

i. $\sin(x) \leq 1/2$

ii. $-1 < \tan(x) < 1$

iii. $2\cos(x) + 1 > 0$

iv. $\sin(x) > \cos(x)$

(c) Find the exact value of each expression:

i. $\tan^{-1}(\sqrt{3})$

ii. $\arcsin(1)$

iii. $\sin\left(\sin^{-1}\left(\frac{7}{10}\right)\right)$

iv. $\arcsin(-1/\sqrt{2})$

v. $\tan(\tan^{-1}(325))$

(d) Simplify using a triangle:

i. $\sin(\cos^{-1}(x))$

ii. $\sin(\tan^{-1}(x))$

iii. $\tan(\sin^{-1}(x))$