More on The Number e

Why does the number e come up so often in Calculus? A brief history of the number e can be found at:

`http://www-gap.dcs.st-and.ac.uk/~history/HistTopics/e.html`

From this, the number e comes to us most directly from work on compound interest (for money). That is, Jacob Bernoulli was trying to compute:

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$$

Let us see where this limit comes from. Suppose I have an amount of money, P dollars, that is being compounded annually at 5%. At the end of the first year, we have

$$P + 0.05P = P(1 + 0.05)$$

At the end of the second year, we have

$$[P(1 + 0.05)] + 0.05[P(1 + 0.05)] = P(1 + 0.05)^2$$

and so on. After n years, we have $P(1 + 0.05)^n$. Using this same procedure, we could show that, with an initial amount of money P_0, with interest rate r compounded n times per year, after t years we have the following amount:

$$P_0\left(1+\frac{r}{n}\right)^{nt}$$

Replacing r with 1, we get that the quantity we're interested in is:

$$\left(1+\frac{1}{n}\right)^n$$

which is the portion depending on the number of compoundings per year. What happens in the case of *continuous* compounding? This has applications in nature, where we might be modeling the growth of a tree, for example. The quantity becomes:

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$$

DEFINITION:[1] The number e is defined to be:

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n \approx 2.71828182845904523 54...$$

$$e^r = \lim_{n\to\infty}\left(1+\frac{r}{n}\right)^n$$

The number e is not rational- that is, the decimal expansion does not repeat. In calculations, treat e just as you would π. The number e comes up in modeling nature- just as π does.
DEFINITION: The natural logarithm is defined as the inverse of e^x. That is, $a = \ln(b)$ if and only if $\mathrm{e}^a = b$.

[1] The mathematician Leonhard Euler (pronounced Oiler) was the first to assign the letter e to this number.