Calculus I Review Questions

NOTE: You should also look over your old exams, old review sheets, old quizzes and homework.

1. Compare and contrast the three "Value Theorems" of the course. When you would typically use each.

2. List the three things we need to check to see if a function f is continuous at $x = a$.

3. Find the point on the parabola $x + y^2 = 0$ that is closest to the point $(0, -3)$.

4. Write the equation of the line tangent to $x = \sin(2y)$ at $x = 1$.

5. What is the general solution to the differential equation:
$$(y - k)' = c(y - k)$$
where y is a function of t, and c, k are constants?

6. Compute the derivative of y with respect to x:
   (a) $y = \sqrt[3]{2x+1}\sqrt[5]{3x-2}$
   (b) $y = \frac{1}{1+u^2}$, where $u = \frac{1}{1+x^2}$
   (c) $\sqrt[3]{y} + \sqrt[3]{x} = 4xy$
   (d) $\sqrt{x+y} = \sqrt[3]{x-y}$
   (e) $y = \sin(2\cos(3x))$
   (f) $y = (\cos(x))^{2x}$
   (g) $y = (\tan^{-1}(x))^{-1}$
   (h) $y = \sin^{-1}(\cos^{-1}(x))$
   (i) $y = \log_{10}(x^2 - x)$
   (j) $y = x^{x^2+2}$
   (k) $y = e^{\cos(x)} + \sin(5^x)$
   (l) $y = \cot(3x^2 + 5)$
   (m) $y = \sqrt{\sin(\sqrt{x})}$
   (n) $\sqrt{x} + \sqrt[3]{y} = 1$
   (o) $x\tan(y) = y - 1$
   (p) $y = \frac{-2}{\sqrt[4]{t^3}}$, where $t = \ln(x^2)$.
   (q) $y = x3^{-1/x}$

7. Find the local maximums and minimums: $f(x) = x^3 - 3x + 1$ Show your answer is correct by using both the first derivative test and the second derivative test.

8. Compute the limit, if it exists. You may use any method (except a numerical table).
   (a) $\lim_{x\to 0} \frac{x - \sin(x)}{x^3}$
   (b) $\lim_{x\to 0} \frac{1 - e^{-2x}}{\sec(x)}$
   (c) $\lim_{x\to 4^+} \frac{x-4}{|x-4|}$
   (d) $\lim_{x\to -\infty} \sqrt{\frac{2x^2-1}{x+8x^2}}$
   (e) $\lim_{x\to\infty} \sqrt{x^2+x+1} - \sqrt{x^2-x}$
   (f) $\lim_{h\to 0} \frac{(1+h)^{-2}-1}{h}$
   (g) $\lim_{x\to\infty} x^3 e^{-x^2}$
   (h) $\lim_{x\to 1} \frac{x^{1000}-1}{x-1}$
   (i) $\lim_{x\to 0} \frac{x}{\tan^{-1}(4x)}$
   (j) $\lim_{x\to 1} x^{\frac{1}{1-x}}$

9. Determine all vertical/horizontal asymptotes and critical points of $f(x) = \frac{2x^2}{x^2-x-2}$

10. Find values of m and b so that (1) f is continuous, and (2) f is differentiable.
$$f(x) = \begin{cases} x^2 & \text{if } x \le 2 \\ mx + b & \text{if } x > 2 \end{cases}$$

11. Find the local and global extreme values of $f(x) = \frac{x}{x^2+x+1}$ on the interval $[-2, 0]$.

12. Suppose f is differentiable so that:
$$f(1) = 1,\ f(2) = 2,\ f'(1) = 1\ f'(2) = 2$$
If $g(x) = f(x^3 + f(x^2))$, evaluate $g'(1)$.

13. Let $x^2y + a^2xy + \lambda y^2 = 0$
   (a) Let a and λ be constants, and let y be a function of x. Calculate $\frac{dy}{dx}$:
   (b) Let x and y be constants, and let a be a function of λ. Calculate $\frac{da}{d\lambda}$:

14. Show that $x^4 + 4x + c = 0$ has at most one solution in the interval $[-1, 1]$.

15. True or False, and give a short explanation.
   (a) If f has an absolute minimum at c, then $f'(c) = 0$.
   (b) If $f(x)$ is decreasing and $g(x)$ is decreasing, then $f(x)g(x)$ is decreasing.

(c) If f is differentiable, then

$$\frac{d}{dx}\sqrt{f(x)} = \frac{f'(x)}{2\sqrt{f(x)}}$$

(d) $\frac{d}{dx}(10^x) = x10^{x-1}$

(e) If $f'(x)$ exists and is nonzero for all x, then $f(1) \neq f(0)$.

(f) If $y = ax + b$, then $\frac{dy}{da} = x$

(g) If $2x + 1 \leq f(x) \leq x^2 + 2$ for all x, then $\lim_{x\to 1} f(x) = 3$.

(h) If $f'(r)$ exists, then

$$\lim_{x\to r} f(x) = f(r)$$

(i) If f and g are differentiable, then:

$$\frac{d}{\mathrm{dx}}(f(g(x)) = f'(x)g'(x)$$

(j) If $f(x) = x^2$, then the equation of the tangent line at $x = 3$ is: $y - 9 = 2x(x - 3)$

(k) $\displaystyle\lim_{\theta\to\frac{\pi}{3}} \frac{\cos(\theta) - \frac{1}{2}}{\theta - \frac{\pi}{3}} = -\sin\left(\frac{\pi}{3}\right)$

(l) There is no solution to $\mathrm{e}^x = 0$

(m) $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right) = \frac{2\pi}{3}$

(n) $5^{\log_5(2x)} = 2x$, for $x > 0$.

(o) $\frac{d}{dx}\ln(|x|) = \frac{1}{x}$, for all $x \neq 0$.

(p) $\frac{d}{dx}10^x = x10^{x-1}$

(q) If $x > 0$, then $(\ln(x))^6 = 6\ln(x)$

16. Find the domain of $\ln(x - x^2)$:

17. Find the value of c guaranteed by the Mean Value Theorem, if $f(x) = \frac{x}{x+2}$ on the interval $[1, 4]$.

18. Given that the graph of f passes through the point $(1, 6)$ and the slope of the tangent line at $(x, f(x))$ is $2x + 1$, find $f(2)$.

19. Simplify, using a triangle: $\cos(\tan^{-1}(x))$ (No calculus needed).

20. A fly is crawling from left to right along the curve $y = 8 - x^2$, and a spider is sitting at $(4, 0)$. At what point along the curve does the spider first see the fly?

21. Compute the limit, without using L'Hospital's Rule. $\displaystyle\lim_{x\to 7}\frac{\sqrt{x+2} - 3}{x - 7}$

22. For what value(s) of c does $f(x) = cx^4 - 2x^2 + 1$ have both a local maximum and a local minimum?

23. If $f(x) = \sqrt{1 - 2x}$, determine $f'(x)$ *by using the definition of the derivative.*

24. A *point of inflection* for a function f is the x value for which $f''(x)$ changes sign (either from positive to negative or vice versa).

    Find constants a and b so that $(1, 6)$ is an inflection point for $y = x^3 + ax^2 + bx + 1$.

    Hint: The IVT might come in handy

25. Suppose that $F(x) = f(g(x))$ and $g(3) = 6$, $g'(3) = 4$, $f(3) = 2$ and $f'(6) = 7$. Find $F'(3)$.

26. Find the dimensions of the rectangle of largest area that has its base on the $x-$axis and the other two vertices on the parabola $y = 8 - x^2$.

27. Let $G(x) = h(\sqrt{x})$. Then where is G differentiable? Find $G'(x)$.

28. If position is given by: $f(t) = t^4 - 2t^3 + 2$, find the times when the acceleration is zero. Then compute the velocity at these times.

29. If $y = \sqrt{5t - 1}$, compute y'''.

30. If $f(x) = (2 - 3x)^{-1/2}$, find $f(0)$, $f'(0)$, $f''(0)$.

31. Car A is traveling west at 50 mi/h, and car B is traveling north at 60 mi/h. Both are headed for the intersection between the two roads. At what rate are the cars approaching each other when car A is 0.3 mi and car B is 0.4 mi from the intersection?

32. Find the linearization of $f(x) = \sqrt{1 - x}$ at $x = 0$.

33. Find $f(x)$, if $f''(x) = t + \sqrt{t}$, and $f(1) = 1$, $f'(1) = 2$.

34. Find $f'(x)$ directly from the definition of the derivative (using limits and without L'Hospital's rule):

    (a) $f(x) = \sqrt{3 - 5x}$
    (b) $f(x) = x^2$
    (c) $f(x) = x^{-1}$

35. If $f(0) = 0$, and $f'(0) = 2$, find the derivative of $f(f(f(f(x))))$ at $x = 0$.

36. Differentiate:

$$f(x) = \begin{cases} \sqrt{x} & \text{if } x \geq 0 \\ \sqrt{-x} & \text{if } x < 0 \end{cases}$$

Is f differentiable at $x = 0$? Explain.

37. $f(x) = |\ln(x)|$. Find $f'(x)$.

38. $f(x) = x e^{g(\sqrt{x})}$. Find $f'(x)$.

39. Find a formula for dy/dx: $x^2 + xy + y^3 = 0$.

40. Show that 5 is a critical number of $g(x) = 2+(x-5)^3$, but that g does not have a local extremum there.

41. Find the general antiderivative:

    (a) $f(x) = 4 - x^2 + 3e^x$

    (b) $f(x) = \frac{3}{x^2} + \frac{2}{x} + 1$

    (c) $f(x) = \frac{1+x}{\sqrt{x}}$

42. Find the slope of the tangent line to the following at the point (3,4): $x^2 + \sqrt{y}x + y^2 = 31$

43. Find the critical values: $f(x) = |x^2 - x|$

44. Does there exist a function f so that $f(0) = -1$, $f(2) = 4$, and $f'(x) \leq 2$ for all x?

45. Find a function f so that $f'(x) = x^3$ and $x+y = 0$ is tangent to the graph of f.

46. Find dy if $y = \sqrt{1-x}$ and evaluate dy if $x = 0$ and $dx = 0.02$. Compare your answer to Δy

47. Fill in the question marks: If f'' is positive on an interval, then f' is ? and f is ?.

48. If $f(x) = x - \cos(x)$, x is in $[0, 2\pi]$, then find the value(s) of x for which

    (a) $f(x)$ is greatest and least.

    (b) $f(x)$ is increasing most rapidly.

    (c) The slopes of the lines tangent to the graph of f are increasing most rapidly.

49. Show there is *exactly* one solution to: $\ln(x) = 3 - x$.

50. Sketch the graph of a function that satisfies all of the given conditions:

$$f(1) = 5 \qquad f(4) = 2 \qquad f'(1) = f'(4) = 0$$
$$\lim_{x\to 2^+} f(x) = \infty, \quad \lim_{x\to 2^-} f(x) = 3 \qquad f(2) = 4$$

51. If $s^2 t + t^3 = 1$, find $\frac{dt}{ds}$ and $\frac{ds}{dt}$.

52. Rewrite the function as a piecewise defined function (which gets rid of the absolute value signs):

$$f(x) = \frac{|3x+2|}{3x+2} \qquad f(x) = \left|\frac{x-2}{(x+1)(x+2)}\right|$$

53. Find all values of c and d so that f is continuous at all real numbers:

$$f(x) = \begin{cases} 2x^2 - 1 & \text{if } x < 0 \\ cx + d & \text{if } 0 \leq x \leq 1 \\ \sqrt{x+3} & \text{if } x > 1 \end{cases}$$

54. Where is f continuous? $f(x) = \sqrt{\frac{x-1}{x^2-4}}$

55. A tank contains 20 kg of salt dissolved in 5000 liters of water. Brine that contains 0.03 kg of salt per liter of water enters the tank at a rate of 25 liters per minute. The solution is kept thoroughly mixed and drains from the tank at the same rate. How much salt remains in the tank after half an hour?

56. A water tank in the shape of an inverted cone with a circular base has a base radius of 2 meters and a height of 4 meters. If water is being pumped into the tank at a rate of 2 cubic meters per minute, find the rate at which the water level is rising when the water is 3 meters deep. ($V = \frac{1}{3}\pi r^2 h$)

57. A ladder 10 feet long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 foot per second, how fast is the top of the ladder slideing down the wall when the bottom of the ladder is 6 feet from the wall?

58. If a snowball melts so that its surface area decreases at a rate of 1 square centimeter per minute, find the rate at which the diameter decreases when the diameter is 10 cm. (The surface area is $A = 4\pi r^2$)

59. A tank contains 1000 liters of brine with 15 kg of dissolved salt. Pure water enters the tank at a rate of 10 liters per minute. The solution is kept thoroughly mixed and drains from the tank at the same rate. How much salt is in the tank after t minutes? Will the tank ever have zero salt?