M125
Fall 2003

Quiz 1

SOLUTIONS

Show all your work!

1. Find the domain: $h(x) = \dfrac{x}{\sqrt{9-x^2}}$

   The domain is the set of all x for which this function is defined. There will be problems if the denominator is zero, and also where $9 - x^2$ is negative. Therefore, we solve for x:

   $$9 - x^2 > 0$$

   To do this, we set up a sign chart with the factors $(3-x)(3+x)$ on the left side. This separates the number line into three parts:

   $$x < -3, \quad -3 < x < 3, \quad x > 3$$

   Putting a test point into each region tells us that the domain is the middle interval,

   $$-3 < x < 3$$

2. For the function $f(x) = x^2$, find and simplify the expression: $\dfrac{f(x+h) - f(x)}{h}$

   $$\frac{(x+h)^2 - x^2}{h} = \frac{x^2 + 2xh + h^2 - x^2}{h} = \frac{2xh + h^2}{h} = 2x + h$$

3. Explain why the following limit does not exist: $\lim\limits_{x\to 3} \dfrac{|x-3|}{x-3}$

   We rewrite the expression without the absolute value signs:

   $$\frac{|x+3|}{x+3} = \begin{cases} \frac{x+3}{x+3}, & \text{if } x \geq -3 \\ \frac{-(x+3)}{x+3}, & \text{if } x < -3 \end{cases} = \begin{cases} 1, & \text{if } x \geq -3 \\ -1, & \text{if } x < -3 \end{cases}$$

   So the limit as $x \to 3$ does not exist overall, since the limit is 1 as we approach from the right, and -1 as we approach from the left.

4. Find the point of intersection of the lines with equations:

   $$y = -2x, \qquad y = \frac{3}{4}x - 2$$

   Substitute:

   $$-2x = \frac{3}{4}x - 2 \Rightarrow 2 = (2 + 3/4)x \Rightarrow 2 = \frac{11}{4}x \Rightarrow \frac{8}{11} = x$$

   and, $y = -2x$, so $y = -\frac{16}{11}$