M125
Fall 2003

Quiz 2

GROUP QUIZ

Show all your work!

1. Use the definition of the derivative to find the equation of the tangent line to $f(x)$ at the given x:

   (a) $f(x) = 1/x, \quad x = 3$

   $$\lim_{h\to 0} \frac{\frac{1}{3+h} - \frac{1}{3}}{h} = \lim_{h\to 0} \frac{3-(3+h)}{3(3+h)h} = \lim_{h\to 0} \frac{-h}{3(3+h)h} = -\frac{1}{9}$$

   This is the slope of the tangent line. A point that the line goes through is (from $x = 3$), $(3, \frac{1}{3})$.
   The equation of the tangent line is $y - \frac{1}{3} = -\frac{1}{9}(x-3)$

   (b) $f(x) = \sqrt{x}, \quad x = 4$

   $$\lim_{h\to 0} \frac{\sqrt{4+h}-\sqrt{4}}{h} = \lim_{h\to 0} \frac{\sqrt{4+h}-2}{h} \cdot \frac{\sqrt{4+h}+2}{\sqrt{4+h}+2} = \lim_{h\to 0} \frac{(4+h)-4}{h\cdot(\sqrt{4+h}+2)} =$$

   $$\lim_{h\to 0} \frac{1}{\sqrt{4+h}+2} = \frac{1}{4}$$

   This is the slope of the tangent line. A point that the line goes through is (from $x = 4$), $(4, 2)$.
   The equation of the tangent line is $y - 2 = \frac{1}{4}(x-2)$

2. Find values of a, c so that

   $$f(x) = \begin{cases} 2x^2, & \text{if } x < a \\ x+3, & \text{if } x > a \\ C & \text{if } x = a \end{cases}$$

   We check the three parts of continuity at $x = a$:

   - The function must be defined at $x = a$: In this case, $f(a) = c$, so this is true for all values of a.
   - The limit must exist at $x = a$.
     Since the function is piecewise defined, take the limits from the right and left separately:

     $$\lim_{x\to a^+} f(x) = \lim_{x\to a^+} x+3 = a+3$$

     and this limit exists for all values of a.
     From the other side,

     $$\lim_{x\to a^-} f(x) = \lim_{x\to a^-} 2x^2 = 2a^2$$

     which exists for all values of a.
     In order for the limit to exist overall, these limits must be the same:

     $$2a^2 = a+3 \Rightarrow 2a^2 - a - 3 = 0 \Rightarrow (2a-3)(a+1) = 0$$

     For the limit to exist, $a = \frac{3}{2}$ or $a = -1$. In the first case that $a = \frac{3}{2}$, this limit is $\frac{9}{2}$. In the case that $a = -1$, the limit is 2.

- Finally, the limit must be equal to $f(a) = c$. Therefore, if $a = \frac{3}{2}$, then $c = \frac{9}{2}$. If $a = -1$, then $c = 2$.

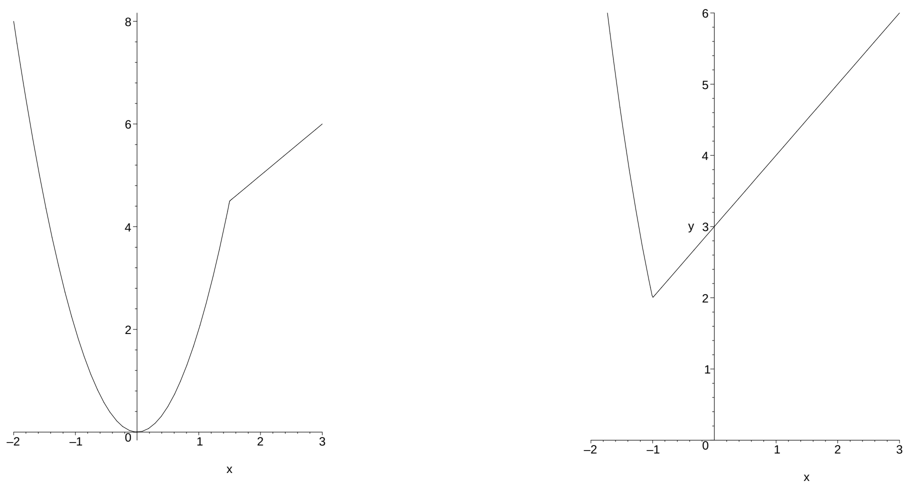

Figure 1: To the right, the result when $a = 3/2$, $c = 9/2$. To the left, the result when $a = -1, c = 2$.