M125
Fall 2003

Quiz 3

SOLUTIONS

1. Using a definition of the derivative, find $f'(x)$ if $f(x) = \sqrt{x}$

$$\lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h} \cdot \frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}} = \lim_{h\to 0} \frac{x+h-x}{h\cdot(\sqrt{x+h}+\sqrt{x})} = \frac{1}{2\sqrt{x}}$$

ALTERNATIVE:

$$\lim_{v\to x} \frac{\sqrt{v}-\sqrt{x}}{v-x} = \lim_{v\to x} \frac{\sqrt{v}-\sqrt{x}}{(\sqrt{v}-\sqrt{x})(\sqrt{v}+\sqrt{x})} = \lim_{v\to x} \frac{1}{\sqrt{v}+\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

For Problems 2 and 3 you may use our shortcut rules for finding the derivative.

2. Let $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 5$. Find all values of x at which f has a horizontal tangent line.

For this problem, we want to realize that a horizontal tangent line takes place where the derivative of the function is zero. Therefore, we want to take the derivative of f, set it to zero, and solve for x:

$$f'(x) = x^2 - x - 2 = (x+1)(x-2)$$

so $f'(x) = 0$ if $x = 2$ or if $x = -1$.

3. Find an equation for the line tangent to $f(x) = 3x + \frac{4}{x}$ when $x = 2$.

We need a point and a slope. The point is obtained by substituting $x = 2$ into f to get the corresponding $y-$ value:

$$f(2) = 6 + 2 = 8$$

so the line goes through the point $(2, 8)$. The slope of the tangent line is given by the derivative and substituting 2 for x. To make the derivative easier, I'll first rewrite f:

$$f(x) = 3x + 4x^{-1} \qquad f'(x) = 3 - 4x^{-2}$$

so the slope is $3 - 4(2^{-2}) = 3 - 1 = 2$.

The line with slope 2 going through $(2, 8)$ is:

$$y - 8 = 2(x - 2)$$